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Mouhamadou Diaby[a,b,*], Abderrahman Iggidr[a], Mamadou Sy[b]

[a] Inria and Université de Lorraine and CNRS. MASAIE, Institut Élie Cartan de Lorraine (IECL, UMR CNRS 7502). ISGMP Bat.A, Ile de Saulcy 57045 Metz Cedex 01, France.
[b] Laboratoire d'Analyse Numérique et Informatique(LANI), Université Gaston Berger de Saint-Louis, UFR SAT BP 234 Saint-Louis, Sénégal.

Abstract

This paper deals with the state estimation for a schistosomiasis infection dynamical model described by a continuous nonlinear system when only the infected human population is measured. The central idea is studied following two major angles. On the one hand, when all the parameters of the model are supposed to be well known, we construct a simple observer and a high-gain Luenberger observer based on a canonical controller form and conceived for the nonlinear dynamics where it is implemented.

On the other hand, when the nonlinear uncertain continuous-time system is in a bounded-error context, we introduce a method for designing a guaranteed interval observer. Numerical simulations are included in order to test the behavior and the performance of the given observers.

Keywords: `high-gain`, `Interval observer`, `Observer`, `Schistosomiasis model`, `State estimation`

1. Introduction

Human schistosomiasis is a behavioral and occupational disease associated with poor human hygiene, insanitary animal husbandry and economic activities. Among human parasitic diseases, schistosomiasis ranks second behind malaria as far as the socio-economic and public health importance in tropical and subtropical areas are concerned. Urinary schistosomiasis, caused by the species Schistosoma haematobium, is common in Africa and the Middle East. The main clinical sign of schistosomiasis infection is haematuria itself caused by the depositions of eggs by an adult female's worms through the bladder by urinary intermediary [1].

*Corresponding author
Email addresses: `diabloss84@yahoo.fr` (Mouhamadou Diaby), `abderrahman.iggidr@inria.fr` (Abderrahman Iggidr), `syndioum@yahoo.fr` (Mamadou Sy)

The most effective form of treatment for infected individuals is the use of the drug praziquantel a drug that kills the worms with high efficiency. Control programs often consist on mass chemotherapy possibly supplemented by snail (intermediate host) control. Since school-age children are the heaviest infected group that suffer the most from morbidity and by that are major sources of infection for the community, school targeted chemotherapy can be then an adequate effective approach to control that morbidity [1, 2].

Schistosomiasis have one of the most complex host-parasite process to model mathematically because of the different steps of growth of larval assumed by the parasite and the requirement of two host elements (definitive human host and intermediate snail hosts) during their life cycle.

Current world-wide interest in the control of schistosomiasis has focused attention upon the intermediate hosts of the causative parasite, since there is general agreement that the most promising method of controlling the disease is to eliminate or greatly reduce the numbers of these vector snails. It is necessary to obtain information about snail populations, whether the information is used for snail-control evaluation, for ecological research, or for the study of transmission potential.

In epidemiology, mathematical models are very often used to describe the dynamic evolution of the diseases. Deterministic Ordinary differential Equations (ODEs) are one of the major modeling tools and are used in our case.

In this paper, we are interested in the estimation problem of the unknown snails population state of a schistosomiasis model whose dynamics are modeled by a continuous time system.

Symbolically, we can write a dynamical system as :

$$\left\{ \begin{array}{rcl} \dot{X}(t) & = & F(X(t)), \\ Y(t) & = & h(X(t)), \end{array} \right. \tag{1}$$

with $X(t) \in \mathbb{R}^n$, $Y(t) \in \mathbb{R}^p$, $p < n$.

If it is possible to have the value of the state at some time t_0 then it is possible to compute $X(t)$ for all $t \geq t_0$ by integrating the differential equation with the initial condition $X(t_0)$. Unfortunately, it is not often possible to measure the whole state at a given time and by the same way to integrate the differential equation because one does not know the initial condition. One can only have a partial information on the state and this partial information is precisely given by $Y(t)$ the output of the system. Therefore we shall show how to use this partial information $Y(t)$ together with the given model in order to have a reliable estimation of the unmeasurable state variables. A state observer is usually employed, in order to accurately reconstruct the state variables of the dynamical system. In the case of linear systems, the observer design theory developed by Luenberger [3], offers a complete and comprehensive answer to the problem. In the field of nonlinear systems, the nonlinear observer design problem is much more challenging and has received a considerable amount of attention in the literature.

An observer for (1) is a dynamical system

$$\begin{cases} \dot{Z}(t) = \hat{F}(Z(t), Y(t)), \\ \hat{X}(t) = L(Z(t), Y(t)) \end{cases} \tag{2}$$

whose task is state estimation. It is expected to provide a dynamical estimate $\hat{X}(t)$ of the state $X(t)$ of the original system. The output is in general a function of the state variable, that is, $Y(t) = h(X(t))$.

One usually requires at least that $\left|\hat{X}(t) - X(t)\right|$ goes to zero as $t \to \infty$. When the convergence of $\hat{X}(t)$ towards $X(t)$ is exponential, the system (1) is an " exponential observer ". More precisely, system (2) is an exponential observer for system (1) if there exists $\lambda > 0$ and $c_0 \geq 0$ such that, for all $t \geq 0$ and for all initial conditions $\left(X(0), \hat{X}(0)\right)$, the corresponding solutions of (1)-(2) satisfy

$$\left|\hat{X}(t) - X(t)\right| \leq e^{-\lambda\, t} \left(\left|\hat{X}(0) - X(0)\right| + c_0\right).$$

The best situation corresponds to the case where $c_0 = 0$. In this situation a good estimate of the real unmeasured state is rapidly obtained. One must notice that we do not need to care about the initial condition of the observer since the convergence of $\hat{X}(t)$ towards the real state $X(t)$ does not depend on this choice.

There are numerous means to deal with the synthesis of nonlinear observers. The most general method to tackle it is to use a "high-gain method observer " when the functions of the variables are perfectly known in the dynamical model. This means is much more general than "the output injection model " developed in [4, 5, 6, 7], which is applied to a very special class of systems only.

If it happens that some functions of the variables are partially known in the dynamical model but bounded with a priori known bounds, we can define a bounded error observer giving $\hat{X}(t)$ with $\left|\hat{X}(t) - X(t)\right|$ bounded by a "reasonable" positive real constant (depending on the uncertainty), "reasonable" meaning that this constant is small with respect to the measurement errors as developed in [8].

This paper shows out first a high-gain observer for a reduced nonlinear model of schistosomiasis as proposed by Allen [9]. This high-gain observer method, has been initiated in [10, 11, 12]. However, the convergence of this kind of observers is difficult to prove (because of the global Lipshitz condition). So, we propose a simpler observer whose convergence analysis is studied. This nonlinear observer design does not require Lipschitz extension of functions and change of coordinates for the system contrary to the high-gain observer.

In the second part, we will present an interval observer design to handle the already mentioned uncertainties of the model parameters. The methodology of interval observers has already been studied using a theoretical framework [13, 14], and interval observers have been developed for particular models [15, 16], and have been validated experimentally [14]. In these works the authors address conditions for stability of the interval observer.

The construction of an observer requires some properties of observability and requires essentially the existence of globally defined and globally Lipschitzian change of coordinates.

The paper is organized as follows: In Section 2 we present the biological assumptions that guided the model's structure and the model's equations. In Section 3 we perform a high-gain observer design. Section 4 will point out a simple observer design. Section 5 tackles the guaranteed interval observer construction. Finally Sections 6 and Section 7 will respectively be constituted of the different estimator simulations and the conclusion.

2. Model and assumptions

In this section, the model proposed is a modified version of Allen's model [9]. The main point in the model presented in Allen in a relatively isolated community, based on the model presented in Allen [9] is to take into account an additional mammalian host as well as a competitor snails. The model assumes that hosts population and infection rates are independent of environmental factors. The totality of simplifying assumptions lead one to question the quantitative predictions of the model. However, the qualitative features of the results are in themselves of considerable interest [9]. Here, we ignore competitor snails population. Thus, the total human population size, denoted by $N_H(t)$, is split into susceptible individuals ($X_1(t)$) and infected individuals ($X_2(t)$) so that $N_H(t) = X_1(t) + X_2(t)$, and the total mammal population size, denoted by $N_M(t)$, is also subdivided into susceptible mammals ($X_6(t)$) and infected mammals ($X_7(t)$) so that $N_M(t) = X_6(t) + X_7(t)$. Whereas, the total snails population, denoted by $N_S(t)$, is subdivided into susceptible snail host ($X_3(t)$), infected snails which are not yet shedding cercariae ($X_4(t)$) and infected and shedding snail ($X_5(t)$). Thus $N_S(t) = X_3(t) + X_4(t) + X_5(t)$. We assume that the total time interval considered, T, is not too large so that the infection in the definitive hosts (e.g. human) does not result death. Further, it is assumed that infected snails and infected mammals do not recover from schistosomiasis as their life spans are short in comparison to that for humans. For simplicity, assume that births in each population group (human, snail and alternate host) were allowed to enter only the uninfected populations. Another assumption made is that the latent periods can be ignored in both definitive and intermediate hosts. This means that we disregard the time period between the moment when a cercaria penetrates a definitive host and the moment when the cercaria has grown to a sexually mature parasite. Other assumptions are that the recovery rate of infected intermediate hosts is independent of the length of the infectious period, and that the rate of output of cercariae from an infected intermediate host is constant throughout the period when it remains infected. We furthermore assume that births and deaths were considered to be proportionate to population size .

Thus, the differential equations which govern the disease are :

$$\begin{cases} \dfrac{dX_1}{dt} = & -t_{15}\, X_5\, X_1 + r_{12}\, X_2, \\ \dfrac{dX_2}{dt} = & t_{15}\, X_5\, X_1 - r_{12}\, X_2, \\ \dfrac{dX_3}{dt} = & b_3\,(X_3 + X_4 + X_5) - t_{32}\, X_2\, X_3 - d_3\, X_3 - t_{37}\, X_3\, X_7, \\ \dfrac{dX_4}{dt} = & t_{32}\, X_2\, X_3 + t_{37}\, X_3\, X_7 - d_4\, X_4 - r_{54}\, X_4, \\ \dfrac{dX_5}{dt} = & r_{54}\, X_4 - d_5\, X_5, \\ \dfrac{dX_6}{dt} = & b_6(X_6 + X_7) - t_{65}\, X_5\, X_6 - d_6\, X_6, \\ \dfrac{dX_7}{dt} = & t_{65}\, X_5\, X_6 - d_7 X_7. \end{cases} \tag{3}$$

Where :

- t_{15} = transmission rate from infected snails to uninfected humans,
- t_{32} = transmission rate from infected humans to uninfected snails,
- t_{37} = transmission rate from infected mammals to susceptible snail,
- t_{65} = transmission rate from infected snails to susceptible mammals.

Birth and death rates for the various sub-populations are denoted by b_i et d_i, respectively, for $i = 1, 2, ..., 7$. Also, r_{12} is the treatment rate of infected human population and r_{54} denotes the rate that the latent snail population X_4 becomes shedding X_5.

It is assumed for simplicity that $b_1 = d_1 = d_2$, $b_3 = d_3 = d_4 = d_5$, and $b_6 = d_6 = d_7$. Then all total populations are constant thanks to the assumptions on b_i and d_i. This is a technical assumption aimed to reduce the complexity of the mathematical analysis, but is not always a plausible approximation. This assumption of a constant population size is also realistic when modeling a disease over many years if the births are approximately balanced by the natural deaths. More general models can be constructed and analyzed, but our goal here is to show what may be deduced from simple model. Furthermore, this simple model has additional value as it is based on models that include more detailed structure.

We introduce the proportions of the snails and mammals sub-populations:

$$x_i = \frac{X_i}{N_S} \text{ for } i = 3, 4, 5, \text{ and } x_i = \frac{X_i}{N_M} \text{ for } i = 6, 7.$$

Using the number of infected humans and the proportions of the other sub-populations and the fact that $X_1 = N_H - X_2$, $x_3 + x_4 + x_5 = 1$, and $x_6 + x_7 = 1$, system (3) reduces to one of the following equivalent systems (4) and (5). The first one using X_2, x_3, x_5, and x_7 is given by:

$$\left\{\begin{array}{rcl} \dfrac{d\,X_2}{dt} & = & t_{15}\,(N_H - X_2)\,N_S\,x_5\, - r_{12}\,X_2, \\ \dfrac{d\,x_3}{dt} & = & b_3 - (t_{32}\,X_2 + t_{37}\,N_M\,x_7 + b_3)\,\,x_3, \\ \dfrac{d\,x_5}{dt} & = & r_{54}\,\,(1 - x_3 - x_5) - b_3\,x_5, \\ \dfrac{d\,x_7}{dt} & = & t_{65}\,N_S\,x_5\,\,(1 - x_7) - b_6\,x_7. \end{array}\right. \tag{4}$$

The second one using X_2, x_4, x_5, and x_7 is given by:

$$\left\{\begin{array}{rcl} \dfrac{d\,X_2}{dt} & = & t_{15}\,(N_H - X_2)\,N_S\,x_5\, - r_{12}\,X_2, \\ \dfrac{d\,x_4}{dt} & = & (t_{32}\,X_2 + t_{37}\,N_M\,x_7)\,\,(1 - x_4 - x_5) - (b_3 + r_{54})\,x_4, \\ \dfrac{d\,x_5}{dt} & = & r_{54}\,x_4 - b_3\,x_5, \\ \dfrac{d\,x_7}{dt} & = & t_{65}\,N_S\,x_5\,\,(1 - x_7) - b_6\,x_7. \end{array}\right. \tag{5}$$

To compute $X(t) = (X_2(t), x_4(t), x_5(t), x_7(t))$ at time t using equations 5, we need to know the value of the state $X(t_0)$ at a given time $t_0 < t$. However, this is not possible in general. To make the model useful, we have to find how to estimate the unknown value $X(t)$. This is the main concern we will address in this paper. To achieve this goal, we will use a tool from control theory called observer. What is noticeable in this model is that the state of snails and mammalians are not available for the measurement in so far as the only available information at time t is the value of the infected human population. This means that it is possible to detect through clinical signs the number of infected people at a given time t (it is provided by health department). Then the measurable output is $y(t) = X_2(t)$.

3. A high-gain observer for a schistosomiasis model

We construct a high-gain observer for our system using the techniques developed in [10, 12]. The high-gain observer construction involves some complicated computations [11, 10, 17]. Since systems (4) and (5) are equivalent, we shall use system (4) which is more adapted to the high-gain observer construction.

Let us denote by $x(t) = (X_2(t), x_3(t), x_5(t), x_7(t))$ the state vector of system (4), f the vector field defining the dynamics of the system (4), and h the output function, that is $y(t) = h(x(t)) = X_2(t)$, and

$$f = \begin{pmatrix} t_{15}\,(N_H - X_2)\,N_S\,x_5\; - r_{12}\,X_2 \\ -\,(t_{32}\,X_2 + t_{37}\,N_M\,x_7 + b_3)\;x_3 + b_3 \\ r_{54}\;(1 - x_3 - x_5) - b_3\,x_5 \\ t_{65}\,N_S\,x_5\;(1 - x_7) - b_6\,x_7 \end{pmatrix}.$$

To construct a high-gain observer for (4), one has to perform a change of coordinates in order to write the system in a simpler form. This usually done by using the output function together with its time derivatives.

Let Φ be the function $\Phi : \mathring{\mathcal{D}} \to \mathbb{R}^4$ ($\mathring{\mathcal{D}}$ is the interior of $\mathcal{D}$) defined as follows:

$$\Phi(x) = \begin{pmatrix} h(x) \\ \\ L_f h(x) \\ \\ L_f^2 h(x) \\ \\ L_f^3 h(x) \end{pmatrix},$$

where L_f denotes the Lie derivative operator with respect to the vector field f. Thus

$$\Phi(x) = \begin{pmatrix} X_2 \\ \\ -r_{12}X_2 + N_S t_{15}\,(N_H - X_2)\,x_5 \\ \\ N_S t_{15}\,(N_H - X_2)\,(r_{54}\,(1 - x_3 - x_5) - b_3 x_5) \\ +\,(-r_{12} - N_S t_{15} x_5)\,(-r_{12}X_2 + N_S t_{15}\,(N_H - X_2)\,x_5) \\ \\ (-r_{54} + r_{54}x_3 + b_3 x_5 + r_{54}x_5) \\ (N_S t_{15}\,(b_3\,(N_H - X_2) + N_H\,(r_{12} + r_{54} + 2N_S t_{15} x_5) - X_2\,(2r_{12} + r_{54} + 2N_S t_{15} x_5))) \\ -\,(r_{12}X_2 - N_H N_S t_{15} x_5 + N_S t_{15} X_2 x_5) \\ \left(r_{12}^2 + 2N_S r_{12} t_{15} x_5 + N_S t_{15}\,(r_{54}\,(-1 + x_3 + x_5) + x_5\,(b_3 + N_S t_{15} x_5))\right) \\ +N_S r_{54} t_{15}\,(N_H - X_2)\,(-b_3 + b_3 x_3 + t_{32} X_2 x_3 + N_M t_{37} x_3 x_7) \end{pmatrix}.$$

The Jacobian of Φ can be written:

$$\frac{d\,\Phi}{d\,x} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -r_{12} - N_S\,t_{15}\,x_5 & 0 & N_S\,t_{15}\,(N_H - X_2) & 0 \\ \alpha_0 & -N_S\,r_{54}\,t_{15}\,(N_H - X_2) & \alpha_1 & 0 \\ \alpha_2 & \alpha_3 & \alpha_4 & N_M\,N_S\,r_{54}\,t_{15}\,t_{37}\,(N_H - X_2)\;x_3 \end{pmatrix},$$

where:

$$\begin{aligned}
\alpha_0 \quad &= (r_{12} + N_S\, t_{15}\, x_5)^{\,2} + N_S\, t_{15}\, (b_3 x_5 + r_{54}\,(-1 + x_3 + x_5)) \\[1ex]
\alpha_1 \quad &= -N_S\, t_{15}\ (b_3\,(N_H - X_2) + N_H\,(r_{12} + r_{54} + 2\, N_S\, t_{15}\, x_5) - X_2\,(2\, r_{12} + r_{54} + 2\, N_S t_{15}\, x_5)) \\[1ex]
\alpha_2 \quad &= N_S\, r_{54}\, t_{15}\, t_{32}\ (N_H - X_2)\ x_3 + N_S\, t_{15}\ (b_3 + 2\, r_{12} + r_{54} + 2\, N_S\, t_{15}\, x_5) + (-r_{12} - N_S t_{15} x_5) \\
&\quad (-b_3 x_5 - r_{54}\,(-1 + x_3 + x_5))\left((r_{12} + N_S\, t_{15} x_5)^{\,2} + N_S\, t_{15}\ (b_3 x_5 + r_{54}\,(-1 + x_3 + x_5))\right) \\
&\quad + N_S\, r_{54}\, t_{15}\,(b_3 - x_3\,(b_3 + t_{32}\, X_2 + N_M\, t_{37}\, x_7)) \\[1ex]
\alpha_3 \quad &= N_S\, r_{54}\, t_{15}\ (2\, b_3\ (N_H - X_2) + N_H\ (r_{12} + r_{54} + t_{32}\, X_2 + 3 N_S\, t_{15}\, x_5 + N_M\, t_{37}\, x_7)) \\
&\quad - N_S\, r_{54}\, t_{15}\,(X_2\,(3 r_{12} + r_{54} + t_{32} X_2 + 3\, N_S\, t_{15}\, x_5 + N_M\, t_{37}\, x_7)) \\[1ex]
\alpha_4 \quad &= b_3^2\, N_H\, N_S\, t_{15} + b_3\, N_H\, N_S\, r_{12}\, t_{15} + N_H\, N_S\, r_{12}^2\, t_{15} + 2\, b_3\, N_H N_S\, r_{54}\, t_{15} + N_H\, N_S\, r_{12}\, r_{54}\, t_{15} \\
&\quad + N_H\, N_S\, r_{54}^2\, t_{15} - 3\, N_H\, N_S^2\, r_{54}\, t_{15}^2 - b_3^2\, N_S\, t_{15}\, X_2 - 3\, b_3\, N_S\, r_{12}\, t_{15}\, X_2 - 3\, N_S\, r_{12}^2\, t_{15}\, X_2 \\
&\quad - 2\, b_3\, N_S\, r_{54}\, t_{15}\, X_2 - 3\, N_S\, r_{12}\, r_{54}\, t_{15}\, X_2 - N_S\, r_{54}^2\, t_{15}\, X_2 + 3\, N_S^2\, r_{54}\, t_{15}^2\ + 3\, N_H\, N_S^2\, r_{54}\, t_{15}^2\, x_3 \\
&\quad - 3\, N_S^2\, r_{54}\, t_{15}^2\, X_2\, x_3 + 6\, b_3\, N_H\, N_S^2\, t_{15}^2\, x_5 + 4\, N_H\, N_S^2\, r_{12}\, t_{15}^2\, x_5 + 6\, N_H\, N_S^2\, r_{54}\, t_{15}^2\, x_5 \\
&\quad - 6\, b_3\, N_S^2\, t_{15}^2\, X_2\, x_5 - 6\, N_S^2\, r_{12}\, t_{15}^2\, X_2\, x_5 - 6\, N_S^2\, r_{54}\, t_{15}^2\, X_2\, x_5 + 3\, N_H\, N_S^3\, t_{15}^3\, {}_5^2 - 3\, N_S^3\, t_{15}^3\, X_2\, x_5^2.
\end{aligned}$$

The determinant of $\dfrac{d\,\Phi}{d\,x}$ can be expressed by:

$$\Gamma(X_2, x_3, x_5) = N_M\, N_S^3\, r_{54}^2\, t_{15}^3\, t_{37}\,(N_H - X_2)^{\,3}\, x_3.$$

The Jacobian $\dfrac{d\,\Phi}{d\,x}$ is non-singular in the region $\mathring{\mathcal{D}}$ and moreover $\Phi(x)$ is one-to-one from $\mathring{\mathcal{D}}$ to $\Phi(\mathring{\mathcal{D}})$. So the map Φ is a diffeomorphism from $\mathring{\mathcal{D}}$ to $\Phi(\mathring{\mathcal{D}})$. This implies that the system (4) with the output $y(t) = X_2(t)$ is observable. In the news coordinates defined by $(z_1, z_2, z_3, z_4)^T = z = \Phi(x) = (h(x), L_f h(x), L_f^2 h(x), L_f^3 h(x))^T$, our system can be written in the canonical form as follows:

$$\left\{\begin{array}{lcl}
\dot{z}(t) & = & \underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{A} z(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \Psi(z(t)) \end{pmatrix}, \\[2ex]
y(t) & = & z_1(t) = \underbrace{(1, 0, 0, 0)}_{C}\, z(t),
\end{array}\right. \tag{6}$$

where: $\Psi(z) = L_f^4 h(\Phi^{-1}(z)) = L_f^4 h(x) = \psi(x)$.

The function ψ is smooth (it is a polynomial function of $x = (X_2, x_3, x_5, x_7)$) on the compact set $\mathcal{D}$. Hence, it is globally Lipschitz on $\mathcal{D}$. Therefore it can be extended by $\tilde{\psi}$, a Lipschitz function on $\mathbb{R}^4$ which satisfies $\tilde{\psi}(x) = \psi(x)$, for all $x \in \mathcal{D}$. In the same manner, we define $\tilde{\Psi}$ the Lipschitz prolongation of the function Ψ. So we have the following system (7) defined on the whole space $\mathbb{R}^4$. The restriction of (7) to the domain $\mathcal{D}$ is the system (6):

$$\left\{ \begin{array}{rcl} \dot{z} & = & A\,z + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{\Psi}(z) \end{pmatrix}, \\ y & = & C\,z. \end{array} \right. \tag{7}$$

According to [10], an exponential (high-gain) observer for system (7) is given by

$$\dot{\tilde{z}} = A\,\tilde{z} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{\Psi}(\tilde{z}) \end{pmatrix} + S^{-1}(\theta)\,C^T\,(y - C\;\tilde{z}), \tag{8}$$

where $S(\theta)$ is the solution of $0 = -\theta\,S(\theta) - A^T\,S(\theta) - S(\theta)\,A^T + C^T\,C$ and θ is large enough.

Here, $S(\theta) = \begin{pmatrix} \frac{1}{\theta} & -\frac{1}{\theta^2} & \frac{1}{\theta^3} & -\frac{1}{\theta^4} \\ -\frac{1}{\theta^2} & \frac{2}{\theta^3} & -\frac{3}{\theta^4} & \frac{4}{\theta^5} \\ \frac{1}{\theta^3} & -\frac{3}{\theta^4} & \frac{6}{\theta^5} & -\frac{10}{\theta^6} \\ -\frac{1}{\theta^4} & \frac{4}{\theta^5} & -\frac{10}{\theta^6} & \frac{20}{\theta^7} \end{pmatrix}.$

This observer is particularly simple since it is only a copy of system (7), together with a corrective term depending on θ. For the proof one can see [10].

An observer for the original system (4) can then be given by:

$$\left\{ \begin{array}{l} \dot{\tilde{z}} = A\,\tilde{z} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{\Psi}(\tilde{z}) \end{pmatrix} + S^{-1}(\theta)\,C^T\,(y - C\;\tilde{z}), \\ \hat{x}(t) = \Phi^{-1}(z(t)). \end{array} \right. \tag{9}$$

Or more simply a high-gain observer for the original system (4) can be given by:

$$\dot{\hat{x}} = \tilde{f}(\hat{x}) + \left[\frac{d\,\Phi}{d\,x}\right]^{-1}_{x=\hat{x}} \times S(\theta)^{-1}\,C^T(y - h(\hat{x})). \tag{10}$$

However, the set $\mathcal{D}$ which is positively invariant for system (4) is not necessary positively invariant for the observer (10), and $\Phi(\mathcal{D})$ is not positively invariant for the observer (8). Therefore the expressions $\left[\frac{d\,\Phi}{d\,x}\right]^{-1}_{x=\hat{x}}$ and $\Phi^{-1}(z(t))$ are not well defined in general.

If there exists $\tilde{\Phi}$ a prolongation of the diffeomorphism Φ to the whole space $\mathbb{R}^4$, that is $\tilde{\Phi}$ is a diffeomorphism from $\mathbb{R}^4$ to $\mathbb{R}^4$ whose restriction to $\mathring{\mathcal{D}}$ is Φ, then it will be sufficient to replace Φ by $\tilde{\Phi}$ in (9) and (10) and so all the expressions will be well defined. However, for our system such a prolongation does not exist since $\frac{d\,\Phi}{d\,x}$ is singular on the set $\{X_2 = N_H\} \cup \{x_3 = 0\}$. So instead of working on $\mathcal{D}$, we have to consider first a set $\mathcal{D}_\epsilon \subset \mathcal{D}$ given by $\mathcal{D}_\epsilon = \mathcal{D} \cap \{X_2 < N_H - \epsilon, x_3 > \epsilon\}$. The positive number ϵ has to be chosen in such away that $\mathcal{D}_\epsilon$ is positively invariant for system (4).

On $\{X_2 = N_H - \epsilon\}$, we have

$$\begin{aligned}\frac{d\,X_2}{dt} &= t_{15}\,\epsilon\,N_S\,x_5\ - r_{12}\,(N_H - \epsilon\,)\\ &\leq \epsilon(t_{15}\,N_S + r_{12}) - r_{12}\,N_H \ \text{(since } x_5 \leq 1).\end{aligned}$$

On $\{x_3 = \epsilon\}$, we have

$$\begin{aligned}\frac{d\,x_3}{dt} &= -\,(t_{32}\,X_2 + t_{37}\,N_M\,x_7 + b_3)\ \epsilon + b_3\\ &\geq b_3 - (t_{32}\,N_H + t_{37}\,N_M + b_3)\,\epsilon\ \text{(since } X_2 < N_H \text{ and } x_7 \leq 1).\end{aligned}$$

So we take $\epsilon \leq \min\left\{\dfrac{r_{12}\,N_H}{t_{15}\,N_S + r_{12}}, \dfrac{b_3}{t_{32}\,N_H + t_{37}\,N_M + b_3}\right\}$.

With this choice of ϵ, the map Φ is a diffeomorphism from $\mathcal{D}_\epsilon$ to $\Phi(\mathcal{D}_\epsilon)$. It is then theoretically possible to find a diffeomorphism $\tilde{\Phi}_\epsilon$ from $\mathbb{R}^4$ to $\mathbb{R}^4$ whose restriction to $\mathcal{D}_\epsilon$ is Φ. However, it is generally difficult to give an explicit analytical expression of the extension $\tilde{\Phi}_\epsilon$. Therefore, the simulations will be done without extending the diffeomorphism Φ.

4. A " simple observer" for a schistosomiasis model

Following the high-gain implementation difficulties related in the previous section, we provide a simple observer design. In this section we consider system (5) on the set $\mathcal{D} = [0, N_H] \times [0, 1] \times [0, 1] \times [0, 1]$.

We shall prove that a simple candidate observer for system (5) on the set $\mathcal{D}$

is given by:

$$
\left\{
\begin{aligned}
\frac{d\hat{X}_2}{dt} &= t_{15}\,(N_H - \hat{X}_2)\,N_S\,\hat{x}_5 \; - r_{12}\,\hat{X}_2 \; + L_1\,(y - \hat{X}_2) \\
\frac{d\,\hat{x}_4}{dt} &= \left(t_{32}\,\hat{X}_2 + t_{37}\,N_M\,\hat{x}_7\right)\,(1 - \hat{x}_4 - \hat{x}_5) - (b_3 + r_{54})\,\hat{x}_4, \\
\frac{d\,\hat{x}_5}{dt} &= r_{54}\,\hat{x}_4 - b_3\,\hat{x}_5, \\
\frac{d\,\hat{x}_7}{dt} &= t_{65}\,N_S\,\hat{x}_5\,\,(1 - \hat{x}_7) - b_6\,\hat{x}_7.
\end{aligned}
\right.
\tag{11}
$$

It is remarkable that the set $\mathcal{D} = [0, N_H] \times [0,1] \times [0,1] \times [0,1]$ is a positively invariant compact set for system (11).

This observer is simply a copy of system (5) plus a corrective term given by $L_1\,(y - \hat{X}_2)$. The number L_1 is a constant positive real number that will be chosen in order to ensure the convergence of the estimation error.

We will denote $x(t) = (X_2(t), x_4(t), x_5(t), x_7(t))$ the state vector of system (5), and $\hat{x}(t) = \left(\hat{X}_2(t), \hat{x}_4(t), \hat{x}_5(t), \hat{x}_7(t)\right)$ the state vector of the candidate observer (11). The estimation error is $e(t) = (e_2(t), e_4(t), e_5(t), e_7(t)) = x(t) - \hat{x}(t)$.

We shall make the following assumptions on the model parameters:

Assumption 4.1. $\dfrac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}} < 1, \quad \dfrac{r_{54}}{b_3} \leq \dfrac{1}{2}, \quad \dfrac{N_S\,t_{65}}{b_6} \leq 1$

Proposition 4.1. *Under the assumption 4.1, the system governed by (11) is an exponential observer for the system (5) for L_1 satisfying*

$$
L_1 \geq \max\left(\frac{N_H\,N_S\,t_{15}\,t_{32}}{b_3 + r_{54}} - r_{12}, 0\right),
$$

i.e, there exists a positive real number λ such that for all initial conditions $(\hat{x}(0), x(0)) \in \mathcal{D} \times \mathcal{D}$, one has $|\hat{x}(t) - x(t)| \leq e^{-\lambda\,t}\,|\hat{x}(0) - x(0)|$.

Proof. The estimation error $e(t) = (e_2(t), e_4(t), e_5(t), e_7(t)) = x(t) - \hat{x}(t)$ obeys the following differential equation:

$$
\dot{e} = A_d\,e + f(x) - f(\hat{x}) \tag{12}
$$

where

$$
A_d = \begin{pmatrix} -L_1 - r_{12} & 0 & 0 & 0 \\ 0 & -b_3 - r_{54} & 0 & 0 \\ 0 & 0 & -b_3 & 0 \\ 0 & 0 & 0 & -b_6 \end{pmatrix}, \; f(x) = \begin{pmatrix} t_{15}\,N_S\,x_5\,(N_H - X_2) \\ (t_{32}\,X_2 + t_{37}\,N_M\,x_7)\,(1 - x_4 - x_5) \\ r_{54}\,x_4 \\ t_{65}\,N_S\,x_5(1 - x_7) \end{pmatrix}.
$$

Let $P = \begin{pmatrix} \dfrac{1}{2\,L_1 + 2r_{12}} & 0 & 0 & 0 \\ 0 & \dfrac{1}{2b_3 + 2r_{54}} & 0 & 0 \\ 0 & 0 & \dfrac{1}{2b_3} & 0 \\ 0 & 0 & 0 & \dfrac{1}{2b_6} \end{pmatrix}$ and consider the following candidate Lyapunov function for the error equation (12):

$$V(e) = e^T\,P\,e$$

We can write:

$$f(x) - f(\hat{x}) = \int_0^1 \frac{\partial f}{\partial x}(sx + (1-s)\hat{x})ds\,e = R(e,\hat{x})\,e$$

The explicit expression of the matrix $R(e,\hat{x})$ is given in appendix A.

Therefore $\dot{e} = (A_d + R(e,\hat{x}))\,e$ and then the derivative of $V(e)$ with respect to time along the solutions of the estimation error equation is

$$\dot{V}(e) = e^T\left(P\,A_d + A_d^T\,P + P\,R(e,\hat{x}) + R(e,\hat{x})^T\,P\right)e.$$

Simple computations give

$$\begin{aligned}
\dot{V}(e) = \quad & -e_2^2 - e_4^2 - e_5^2 - e_7^2 \\
& -\frac{t_{15}(e_5+\hat{x}_5)N_S}{L_1+r_{12}}\,e_2^2 - \left(\frac{t_{32}(e_2+\hat{X}_2)}{b_3+r_{54}} + \frac{t_{37}(e_7+\hat{x}_7)N_M}{b_3+r_{54}}\right)e_4^2 - \frac{t_{65}(e_5+\hat{x}_5)N_S}{b_6}\,e_7^2 \\
& +\frac{t_{15}N_S\left(N_H-\hat{X}_2\right)}{L_1+r_{12}}\,e_2e_5 + \left(\frac{r_{54}}{b_3} - \frac{t_{32}(e_2+\hat{X}_2)+t_{37}(e_7+\hat{x}_7)N_M}{b_3+r_{54}}\right)e_4e_5 \\
& +\frac{t_{65}(1-\hat{x}_7)N_S}{b_6}\,e_5e_7 - \frac{(\hat{x}_4+\hat{x}_5-1)\,(e_2t_{32}+e_7t_{37}N_M)}{b_3+r_{54}}\,e_4 \\
\dot{V}(e) = \quad & -e_2^2\left(\frac{t_{15}x_5N_S}{L_1+r_{12}}+1\right) - e_4^2\left(\frac{t_{37}x_7N_M+t_{32}X_2}{b_3+r_{54}}+1\right) - e_5^2 - e_7^2\left(\frac{t_{65}x_5N_S}{b_6}+1\right) \\
& -\frac{t_{32}(\hat{x}_4+\hat{x}_5-1)}{b_3+r_{54}}\,e_2e_4 + \frac{t_{15}N_S\left(N_H-\hat{X}_2\right)}{L_1+r_{12}}\,e_2e_5 + \left(\frac{r_{54}}{b_3} - \frac{t_{37}x_7N_M+t_{32}X_2}{b_3+r_{54}}\right)e_4e_5 \\
& -\frac{t_{37}N_M(\hat{x}_4+\hat{x}_5-1)}{b_3+r_{54}}e_4e_7 + \frac{t_{65}(1-\hat{x}_7)N_S}{b_6}\,e_5e_7
\end{aligned}$$

The expression of $\dot{V}$ can be written:

$$\begin{aligned}
\dot{V}(e) = \quad & -(a_2+1)\,e_2^2 - (a_4+1)\,e_4^2 - e_5^2 - (a_7+1)\,e_7^2 \\
& -b_{24}\,e_2e_4 + b_{45}\,e_4e_5 - b_{47}\,e_4e_7 + b_{57}\,e_5e_7 + b_{25}\,e_2e_5,
\end{aligned}$$

with: $a_2 = \dfrac{x_5 N_S t_{15}}{L_1 + r_{12}}$; $a_4 = \dfrac{X_2 t_{32} + x_7 N_M t_{37}}{b_3 + r_{54}}$; $a_7 = \dfrac{x_5\, N_S t_{65}}{b_6}$;

$$b_{24} = \frac{(-1 + \hat{x}_4 + \hat{x}_5)\, t_{32}}{b_3 + r_{54}};\; b_{45} = \frac{r_{54}}{b_3} - \frac{X_2 t_{32} + x_7 N_M t_{37}}{b_3 + r_{54}};\; b_{47} = \frac{(-1 + \hat{x}_4 + \hat{x}_5) N_M\, t_{37}}{b_3 + r_{54}};$$

$$b_{57} = \frac{(1 - \hat{x}_7)\, N_S t_{65}}{b_7};\; b_{25} = \frac{\left(-\hat{X}_2 + N_H\right) N_S t_{15}}{L_1 + r_{12}}.$$

The derivative of $V(e)$ can be seen as a quadratic form in e_i. Applying the Gauss-Lagrange reduction to this quadratic form leads to:

$$-\dot{V}(e) = (1 + a_2)\ (e_2 + F_2(e_4, e_5))^2 + l_1\ (e_4 + F_4(e_5, e_7))^2 + l_2\ (e_5 + F_5(e_7))^2 + l_3\, e_7^2$$

where l_1, l_2, l_3 are functions of the model parameters, and the F_i are linear forms in their arguments. The exact expressions of the l_i and the F_i are given in Appendix A.

In Appendix A we show that if the parameters satisfy Assumption 4.1, then it is possible to choose a gain L_1 in such a way that all the l_i are positive. This proves that $\dot{V}$ is negative definite which ends the proof. □

5. Design of the interval observers for a schistosomiasis model

The logic of interval observers is to generate estimated bounds that are caused by a lack of reliability in the models of measurements [13, 14]. Here, we intend to explore the possibility of designing interval observers in the case where transmission rate of the model (3) that are t_{15}, t_{32}, t_{37} and t_{65} remain partially known. Moreover, we seek to obtain an estimation even during the transients of the system that is to choose bounds that have been valid since the beginning. Given the uncertain bounds in the model, we are looking for dynamic ones to estimate the variables. This situation resembles many epidemiological models either by the lack of confidence in model parameters calibrated from experimental data, or because of models dynamical simplicity taking into account their complexity.

We will consider in this case that such parameters are bounded by a given positive number rather than given by a single value. The design is based on two points observers which help estimate in real time the lower and upper bound of the vectors state, given the following type of uncertain nonlinear systems:

$$(S) : \begin{cases} \dot{x}(t) &=& Ax(t) + \psi(x, p), \\ y(t) &=& Cx(t), \\ x(t_0) &\in& [x_0] \wedge p \in [p], \end{cases}$$

where A is a matrix of dimension $n \times n$, $t \in [t_0, t_{n_T}]$, $[p] = [p^-, p^+]$ is a real interval vector of $\mathbb{R}^{n_p}$, $\psi \in \mathcal{C}^{k-1}(\mathcal{D} \times [p])$, $\mathcal{D} \times [p] \subseteq \mathbb{R}^{n+n_p}$ is an open set; n, m and n_p are the dimension of respectively the state vector x, the output vector y and the uncertain parameter vector p. We assume that measurements $y_m(t)$ are subject to an unknown but bounded, with known bound, additive error. Thus the feasible domain for measurements are given by the following boxes

$$\mathbb{Y} = [y_m(t) - b, y_m(t) + b],$$

where b is the vector of maximal measurement error.

We suppose that the input is uncertain with known bounds ψ^-, ψ^+ such that:

$$\psi^- \leq \psi \leq \psi^+, \forall t \in \mathbb{R}^+.$$

Remark 5.1. *The inequalities applied to vectors must be considered term by term.*

Under this assumption, we build two asymptotic observers.

Definition 5.1. *Let us consider the system (S). The pair of systems* (S^-, S^+) *with*

$$(S^-) : \begin{cases} \dot{x}^-(t) &= Ax^-(t) + B^-(\psi^-(t), \psi^+(t)), \\ x^-(t_0) &= x_0^-, \end{cases}$$

$$(S^+) : \begin{cases} \dot{x}^+(t) &= Ax^+(t) + B^+(\psi^-(t), \psi^+(t)), \\ x^+(t_0) &= x_0^+, \end{cases}$$

where $x_0^- \leq x_0 \leq x_0^+$ *is an interval estimator for the system (S) if for any compact set* $\mathcal{D}_0 \subset \mathcal{D}$*, the coupled system* (S, S^-, S^+) *verifies for any initial conditions* $x(t_0) \in \mathcal{D}_0$*:*

$$\forall t \geq t_0, \;\; x^-(t) \leq x(t) \leq x^+(t).$$

Function B^+ *(respectively* B^-*) is such that:*

$$B^-(\psi^-(t), \psi^+(t)) \leq B(\psi(t)) \leq B^+(\psi^-(t), \psi^+(t)).$$

We assume that the imprecisely known function ψ can be bounded by a lower and upper Lipschitz function. Moreover, there exists two known functions $\psi^-(\cdot)$ and $\psi^+(\cdot)$ built according to the bounds of $[p]$ and a known number $M < +\infty$ such that:

$$\begin{cases} \forall p \in [p], \;\; \forall x \in \mathcal{D}, \\ \psi^-(x,p) \leq \psi(x,p) \leq \psi^+(x,p), \\ \|\psi^-(x,p) - \psi^+(x,p)\| \leq M. \end{cases} \tag{13}$$

Proposition 5.1. *([14])*

If there exists a gain K, a positive matrix, such as the non-diagonal elements of the matrix $(A - KC)$ are non-negative and Hurwitz with the condition (13) fulfilled, we can propose the interval observer for system (S) the same spirit as for the classical Luenberger approach, provided $x_0^- \leq x_0 \leq x_0^+$:

$$\left\{ \begin{array}{lcl} \dot{x}^+(t) & = & (A - KC)\, x^+(t) + \psi^+(x^-, x^+, p^-, p^+, u(t)) + K\, y_m^+(t), \\ \dot{x}^-(t) & = & (A - KC)\, x^-(t) + \psi^-(x^-, x^+, p^-, p^+, u(t)) + K\, y_m^-(t). \end{array} \right. \tag{14}$$

Remark: It is shown in [14] that the observation error remains positive and with $A - K\,C$ a Hurwitz matrix this error converges.

We consider our model whose dynamics are expressed in (4) and rewriting as follows for some convenience.

$$\left\{ \begin{array}{lcl} \dfrac{d\,X_2}{dt} & = & t_{15}\,(N_H - X_2)\,N_S\,x_5\ - r_{12}\,X_2, \\ \dfrac{d\,x_4}{dt} & = & (t_{32}\,X_2 + t_{37}\,N_M\,x_7)\ (1 - x_4 - x_5) - (b_3 + r_{54})\,x_4, \\ \dfrac{d\,x_5}{dt} & = & r_{54}\,x_4 - b_3\,x_5, \\ \dfrac{d\,x_7}{dt} & = & t_{65}\,N_S\,x_5\ (1 - x_7) - b_6\,x_7. \end{array} \right. \tag{15}$$

We write system (15) in the typical form of (S) where

$$A = \begin{pmatrix} -r_{12} & 0 & 0 & 0 \\ 0 & -(b_3 + r_{54}) & 0 & 0 \\ 0 & r_{54} & -b_3 & 0 \\ 0 & 0 & 0 & -b_6 \end{pmatrix}$$

and

$$\psi = \begin{pmatrix} t_{15}\,(N_H - X_2)\,N_S\,x_5 \\ (t_{32}X_2 + t_{37}N_M x_7)\,(1 - x_4 - x_5) \\ 0 \\ t_{65}N_{\mathrm{S}}x_5\,(1 - x_7) \end{pmatrix}$$

and $C = (1, 0, 0, 0)$.

We assume that the transmission rates t_{15}, t_{32}, t_{37} and t_{65} are unknown but belong to the following intervals $[t_{15}^-, t_{15}^+]$, $[t_{32}^-, t_{32}^+]$, $[t_{37}^-, t_{37}^+]$, $[t_{65}^-, t_{65}^+]$ respectively. We denote $p = (t_{15}, t_{32}, t_{37}, t_{65})$. The initial state variables are also unknown but within the interval vector.

All the components $\psi(\cdot)$ of the vector function are Lipschitz on $\mathcal{D}$ with respect to the state vector for any $p \in [p]$. Then we can consider a Lipschitz extension of $\psi(\cdot)$ on $\mathbb{R}^4$ since $\mathcal{D}$ is positively invariant by the system (15), see for example [18], that we also denote by $\psi(\cdot)$. Moreover, there exists two known functions $\psi^-(\cdot)$ and $\psi^+(\cdot)$ built according to the bounds of $[p]$ such as

$$\psi^- = \begin{pmatrix} t_{15}^- \left(N_H - X_2\right) N_S\, x_5 \\ \left(t_{32}^- X_2 + t_{37}^- N_M x_7\right)\left(1 - x_4 - x_5\right) \\ 0 \\ t_{65}^- N_{\mathrm{S}} x_5 \left(1 - x_7\right) \end{pmatrix}$$

and

$$\psi^+ = \begin{pmatrix} t_{15}^+ \left(N_H - X_2\right) N_S\, x_5 \\ \left(t_{32}^+ X_2 + t_{37}^+ N_M x_7\right)\left(1 - x_4 - x_5\right) \\ 0 \\ t_{65}^+ N_{\mathrm{S}} x_5 \left(1 - x_7\right) \end{pmatrix}.$$

It is not hard to show that there exists a real positive number $M < +\infty$ such that the condition (13) fulfilled (see Appendix B). For the observer gain $K = [l,\ 0,\ 0,\ 0]^T$, where l is a positive real number, the matrix

$$A - KC = \begin{pmatrix} -r_{12} - l & 0 & 0 & 0 \\ 0 & -b_3 - r_{54} & 0 & 0 \\ 0 & r_{54} & -b_3 & 0 \\ 0 & 0 & 0 & -b_6 \end{pmatrix}$$

is Hurwitz.

Therefore, given an interval estimate $([z_1^-, z_1^+], [z_2^-, z_2^+], [z_3^-, z_3^+], [z_4^-, z_4^+])^T$, the estimation in the basis (X_2, x_4, x_5, x_7) is given by

$$\left\{\begin{array}{lcl} \dot{z}_1^+ &=& -(r_{12} + l)\, z_1^+ + t_{15}^+ \left(N_H - z_1^+\right) z_3^+ N_{\mathrm{S}} + l\, y_m^+(t), \\ \dot{z}_2^+ &=& -\left(b_3 + r_{54}\right) z_2^+ + \left(t_{32}^+ z_1^+ + t_{37}^+ N_M z_4^+\right)\left(1 - z_2^+ - z_3^+\right), \\ \dot{z}_3^+ &=& r_{54} z_2^+ - b_3\, z_3^+ \\ \dot{z}_4^+ &=& -b_6 z_4^+ + t_{65}^+ N_{\mathrm{S}}\, z_3^+ \left(1 - z_4^+\right), \\ \dot{z}_1^- &=& -(r_{12} + l)\, z_1^- + t_{15}^- \left(N_H - z_1^-\right) z_3^- N_{\mathrm{S}} + l\, y_m^-(t), \\ \dot{z}_2^- &=& -\left(b_3 + r_{54}\right) z_2^- + \left(t_{32}^- z_1^- + t_{37}^- N_M z_4^-\right)\left(1 - z_2^- - z_3^-\right), \\ \dot{z}_3^- &=& r_{54} z_2^- - b_3\, z_3^-, \\ \dot{z}_4^- &=& -b_6\, z_4^- + t_{65}^- N_{\mathrm{S}}\, z_3^- \left(1 - z_4^-\right). \end{array}\right. \tag{16}$$

6. Simulations

This part consists in showing out some simulation exercises to stress on the efficiency of the proposed observers of system (4) and system (5). The different snails and mammals populations are estimated via infected humans population measurements. In the first place the "simple observer" design was implemented. The population sizes estimation behave satisfactory along the simulations. For the simulation of the high-gain observer we extend the function f that defines the system (4) by continuity in order to make it globally Lipschitz on $\mathbb{R}^4$ in the following way: We denote $\tilde{f}$ the prolongation of f to $\mathbb{R}^4$ and the function π the projection on the domain $\mathcal{D}$ and we construct $\tilde{f} = f \circ \pi$. The extend function $\tilde{f}$ has the same Lipschitz coefficient as f. The projection π is defined as follows: for $x \in \mathbb{R}^4$, $\pi(x) = \overline{x}$ where $\overline{x} \in \mathcal{D}$ is such that $dist(x, \mathcal{D}) = \|x - \overline{x}\|$, i.e., $\overline{x}$ satisfies $\|x - \overline{x}\| = min_{u \in \mathcal{D}} \|u - x\|$. The initial values of state variables in all simulations concerning the high-gain observer are $X_2(0) = 1600$, $x_3(0) = 0.4$, $x_5 = 0.3$, $x_7 = 0.5$, $\hat{X}_2(0) = 2000$, $\hat{x}_3(0) = 0.7$, $\hat{x}_5(0) = 0.1$, $\hat{x}_7(0) = 0.6$.

The parameter values used in the simulation are those estimated and reported in [9]. More specifically, $t_{15} = 2,23 \times 10^{-7}$, $t_{37} = 5,21 \times 10^{-6}$, $t_{32} = 5,23 \times 10^{-6}$, $t_{65} = 2,78 \times 10^{-7}$, $r_{12} = 4,47 \times 10^{-4}$, $r_{54} = 2,50 \times 10^{-2}$, $b_3 = 6,00 \times 10^{-2}$, $d_6 = 2,74 \times 10^{-4}$, $d_3 = 8,86 \times 10^{-3}$, $b_6 = 5.56 \times 10^{-4}$. We choose the following $N_H = 5000$, $N_S = 95000$ and $N_M = 2500$.

With these parameters, the gain chosen is $\theta = 1$ in all the simulations.

Based on Figures 1 to 4, one can see that the high-gain observer performs as expected and that the convergence of the estimates delivered by the high-gain observer is quite fast.

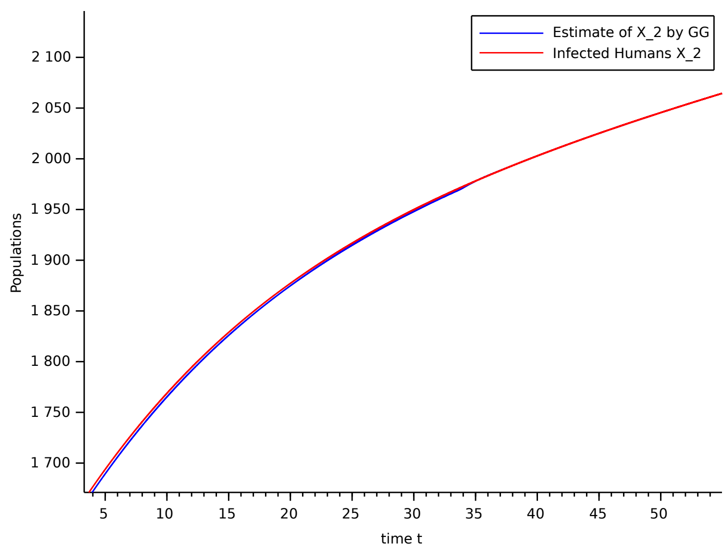

Figure 1: Simulation of system (4) with its high-gain observer (10): $X_2(t)$(red curve) and its estimate $\hat{X}_2(t)$ (blue curve) delivered by the high-gain observer (10) when f is extended

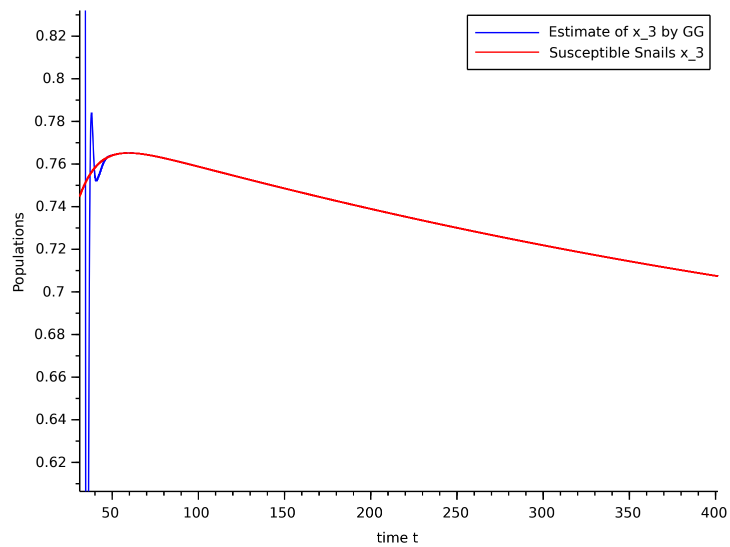

Figure 2: Simulation of system (4) with its high-gain observer (10): $x_3(t)$(red curve) and its estimate $\hat{x}_3(t)$ (blue curve) delivered by the high-gain observer (10) when f is extended

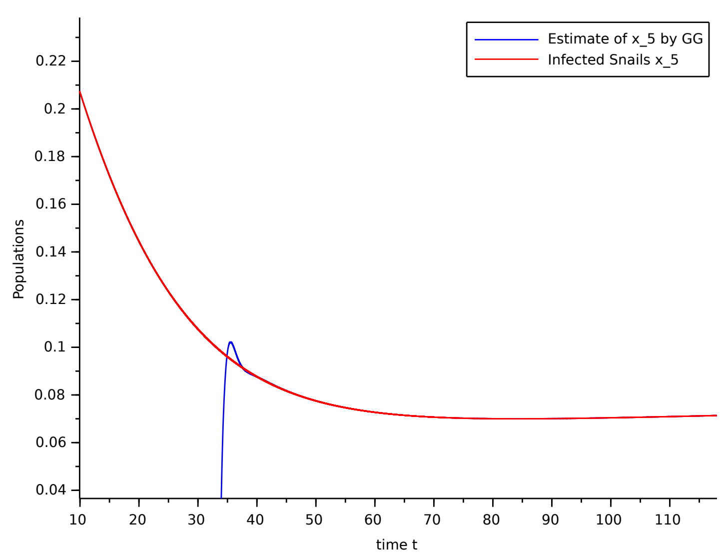

Figure 3: Simulation of system (4) with its high-gain observer (10): $x_5(t)$(red curve) and its estimate $\hat{x}_5(t)$ (blue curve) delivered by the high-gain observer (10) when f is extended

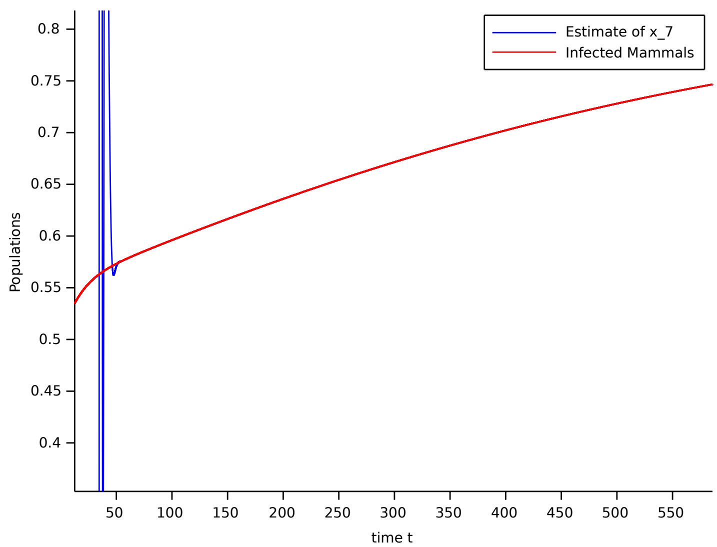

Figure 4: Simulation of system (4) with its high-gain observer (10): $x_7(t)$(red curve) and its estimate $\hat{x}_7(t)$ (blue curve) delivered by the high-gain observer (10) when f is extended

In order to test the robustness of the high-gain observer to noisy measurements, the measurements are vitiated by an additive Gaussian noise (5% of a normal gaussian noise with mean zero and standard deviation one). In Figures 5 to 8 the states variables and their estimates obtained using noisy data of X_2 are shown. Simulations corresponding to free-noise and noisy data have been carried out with the same θ value. The chosen value is the one which provided the best compromise between fast convergence and well noise rejection. In the case of noisy measurements, the choice of relatively high values of θ are to be avoided since they amplify the noise and the obtained estimates may be unusable.

As it is known, the high-gain observer is very sensitive to data noise (one can see [19] and references therein) while our simple observer is less sensitive to data noise. Thus if the output measurements $X_2(t)$ are not good enough, it is appropriate to use the simple observer (11) (see Figures 13 to 16) because we obtain a much less good estimation with the high-gain observer (10) as it is illustrated in Figures 6 to 8.

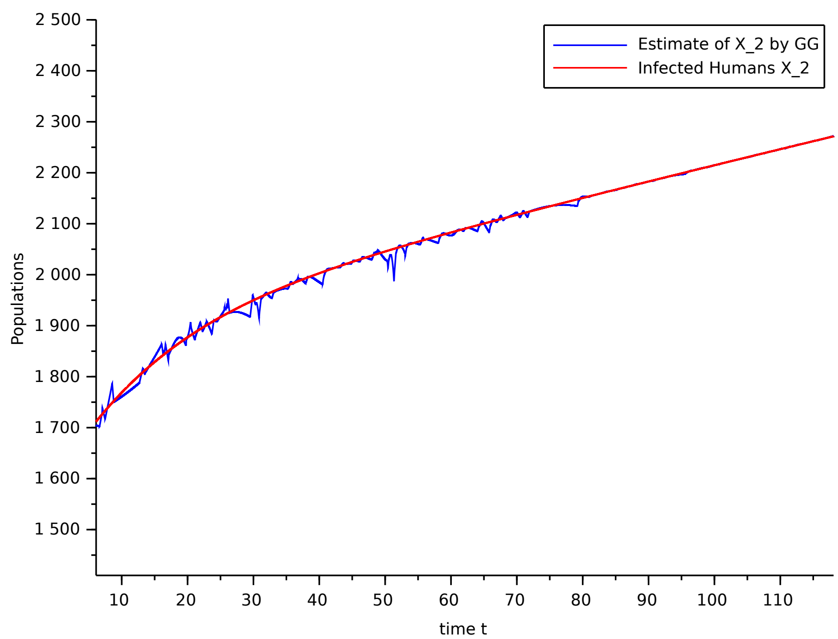

Figure 5: Simulation of system (4) and its high-gain observer (10) when the output measurements are corrupted by noise when f is extended: $X_2(t)$(red curve) and its estimate $\hat{X}_2(t)$ (blue curve) delivered by the high-gain observer (10)

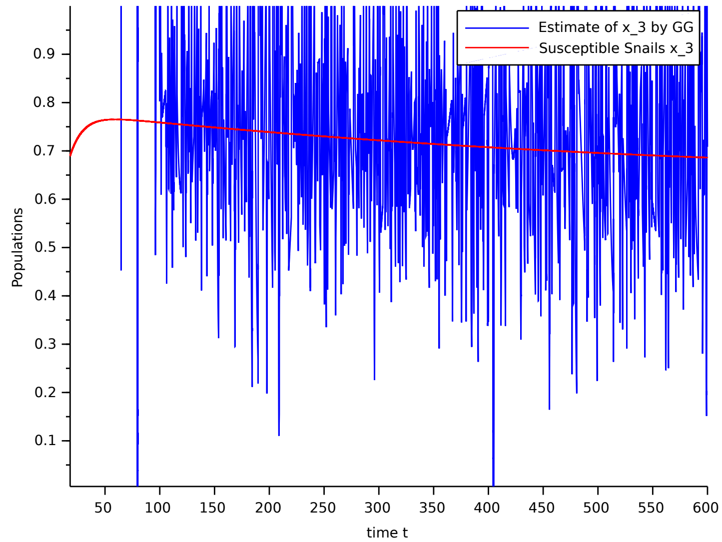

Figure 6: Simulation of system (4) and its high-gain observer (10) when the output measurements are corrupted by noise when f is extended: $x_3(t)$(red curve) and its estimate $\hat{x}_3(t)$ (blue curve) delivered by the high-gain observer (10)

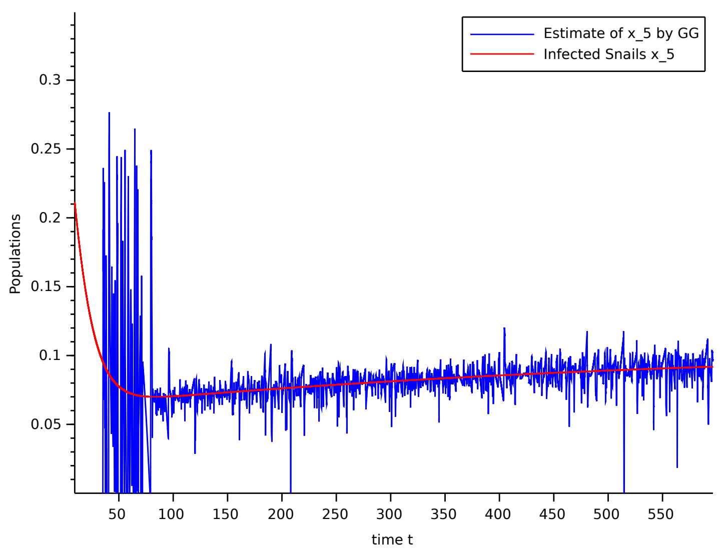

Figure 7: Simulation of system (4) and its high-gain observer (10) when the output measurements are corrupted by noise when f is extended: $x_5(t)$(red curve) and its estimate $\hat{x}_5(t)$ (blue curve) delivered by the high-gain observer (10)

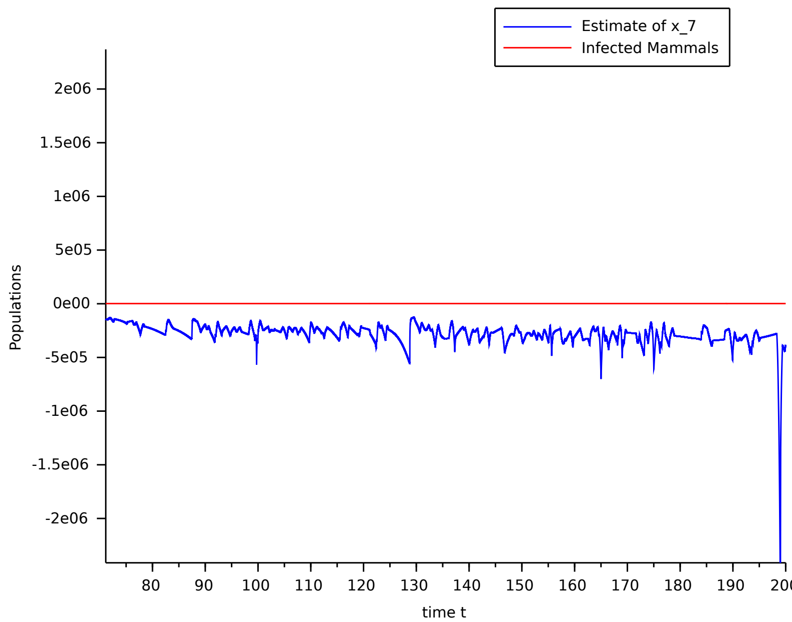

Figure 8: Simulation of system (4) and its high-gain observer (10) when the output measurements are corrupted by noise when f is extended: $x_7(t)$(red curve) and its estimate $\hat{x}_7(t)$ (blue curve) delivered by the high-gain observer (10)

In the following illustrative figures (Figure 9 to Figure 11), we verify the effectiveness of the proposed simple observer defined by system (11).

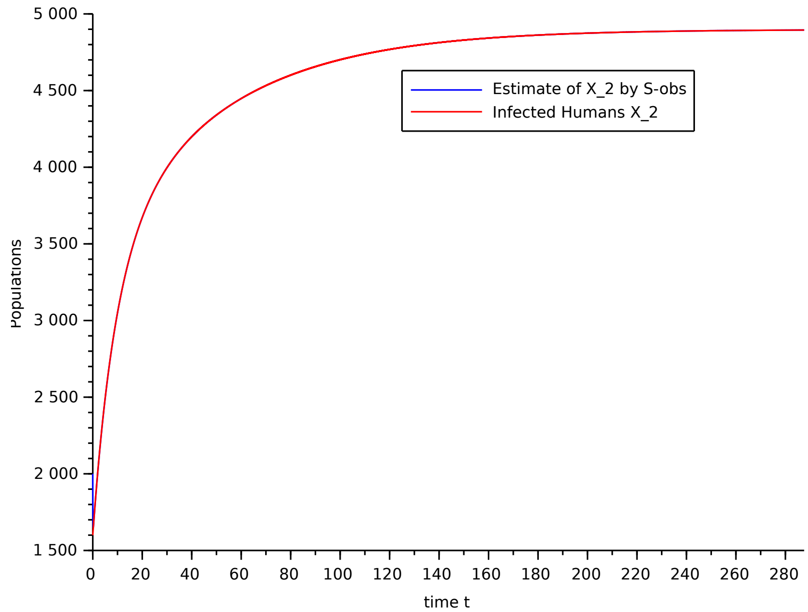

Figure 9: Time evolution of the number of infected humans $X_2(t)$ (red curve) given by (5) and its estimate $\hat{X}_2(t)$ (blue curve) delivered by the simple observer (11)

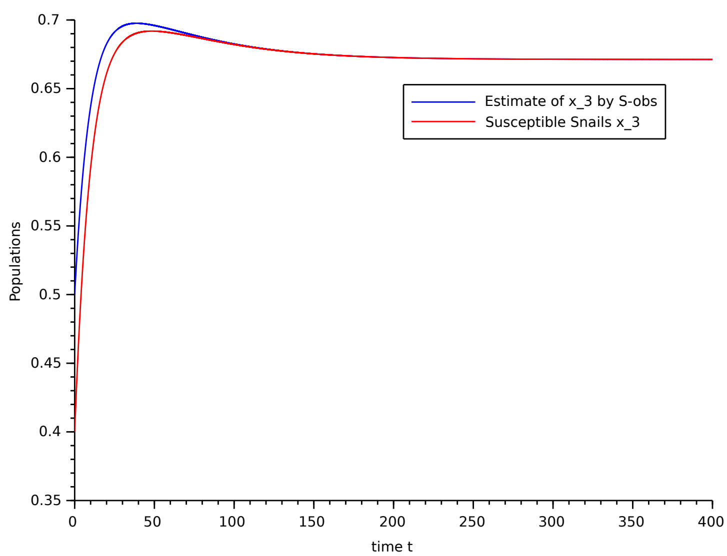

Figure 10: Time evolution of the number of suceptible snails $x_3(t)$ (red curve) given by (5) and its estimate $\hat{x}_3(t)$ (blue curve) delivered by the simple observer (11).

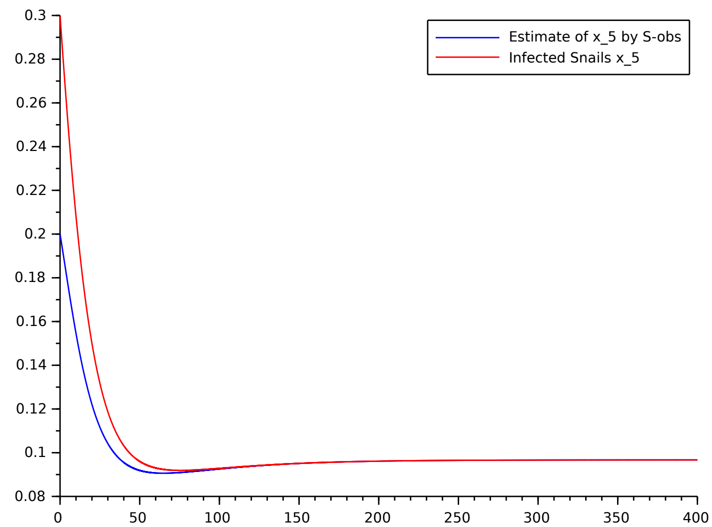

Figure 11: Time evolution of the number of infected snails $x_5(t)$ (red curve) given by (5) and its estimate $\hat{x}_5(t)$ (blue curve) delivered by the simple observer (11).

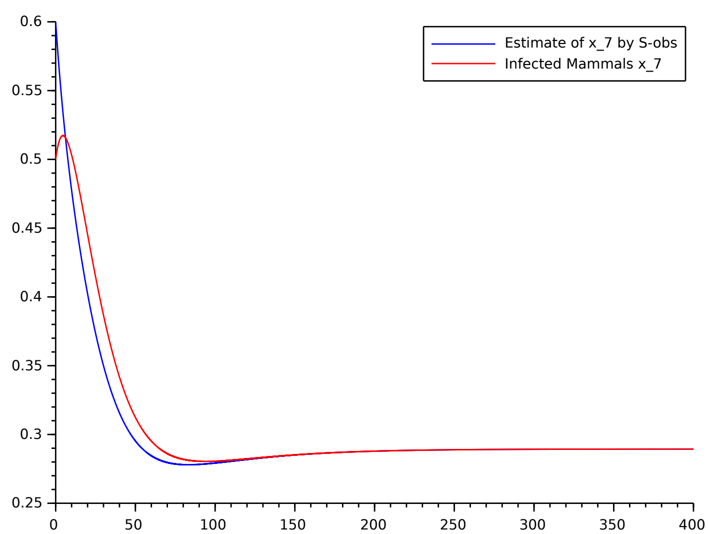

Figure 12: Time evolution of the number of infected mammals $x_7(t)$ (red curve) given by (5) and its estimate $\hat{x}_7(t)$ (blue curve) delivered by the simple observer (11).

To show the effectiveness of the observer in noise compensation, simulation results are included in Figures 13 to 15. We have added to the output measurements $X_2(t)$ of the continuous system, 5% of a normal gaussian noise with mean zero and standard deviation one. These results show that estimation populations are weakly affected by the noise.

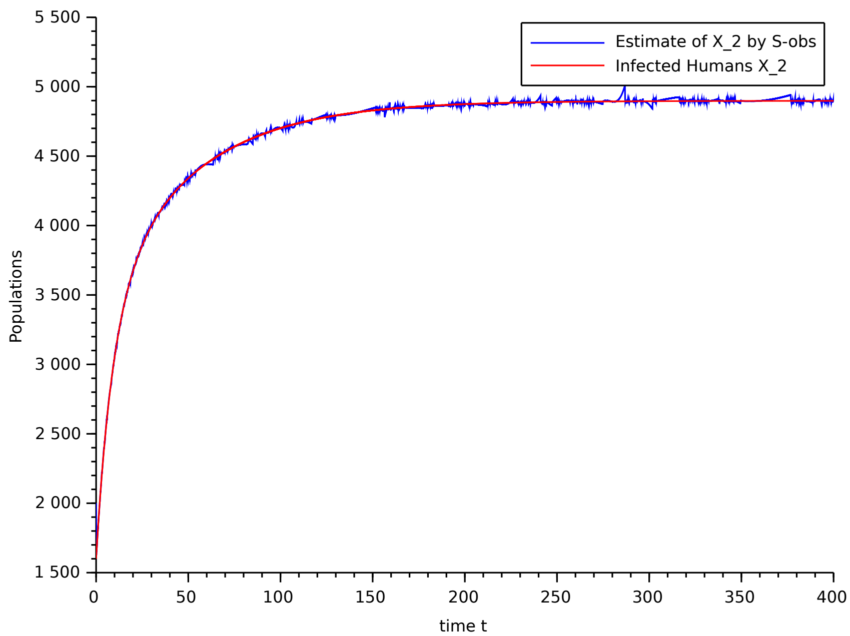

Figure 13: Infected humans population $X_2(t)$ (red curve) and its estimate $\hat{X}_2(t)$ (blue curve) delivered by the simple observer (11) when the output measurements are corrupted by noise.

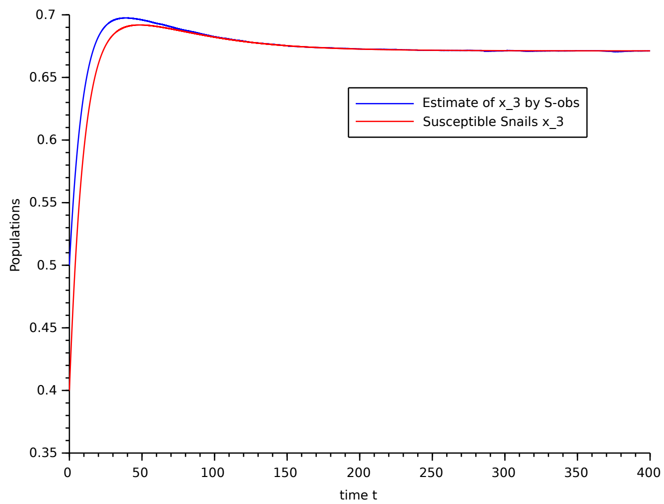

Figure 14: Susceptible snails population $x_3(t)$ (red curve) and its estimate $\hat{x}_3(t)$ (blue curve) delivered by the simple observer (11) when the output measurements are corrupted by noise.

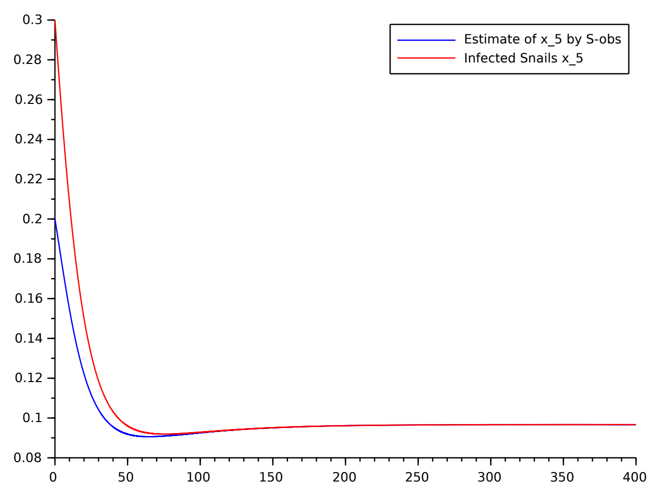

Figure 15: Infected snails population $x_5(t)$ (red curve) and its estimate $\hat{x}_5(t)$ (blue curve) delivered by the simple observer (11) when the output measurements are corrupted by noise.

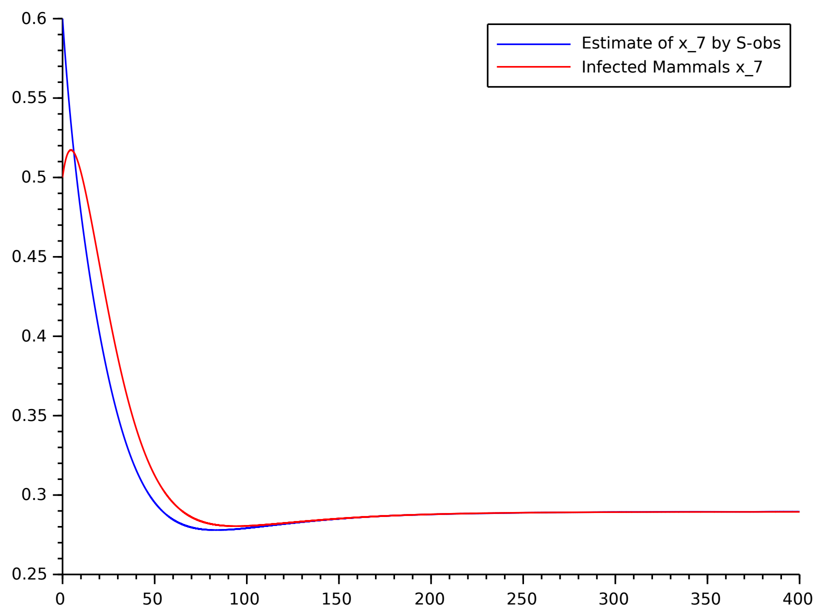

Figure 16: Infected mammals population $x_7(t)$ (red curve) and its estimate $\hat{x}_7(t)$ (blue curve) delivered by the simple observer (11), for system (5) when the output measurements are corrupted by noise.

The time histories of the estimates of the bounds on $X = (X_2, x_3, x_5, x_7)$, the states of the system (3), under the constraint that the transmission rates t_{32}, t_{37}, t_{65} are uncertain, are given in Figure 17, Figure 18, and Figure 19. The partially unknown parameters vector is

$$\begin{aligned} p &= [t_{15}, t_{32}, t_{37}, t_{65}]^T \in [t_{15}^-, t_{15}^+] \times [t_{32}^-, t_{32}^+] \times [t_{38}^-, t_{38}^+] \times [t_{75}^-, t_{75}^+] \\ &= [1.23\,10^{-7}, 2.23\,10^{-7}] \times [0.05\,10^{-7}, 1.05\,10^{-7}] \\ &\quad \times [1.00\,10^{-7}, 2.00\,10^{-7}] \times [0.02\,10^{-7}, 1.02\,10^{-7}]. \end{aligned}$$

Model output is taken as $y(t) = X_2(t)$ and the maximal measurement error is $b = \mp 20\%\, y_m(\infty)$ where $y_m(\infty)$ is a nominal value. A gain vector is taken as $K^T = (3, 0, 0, 0)$. These last figures demonstrate that even with loose initial estimations on each bounds of unmeasured variables, we obtain estimates of the uncertainty intervals with a reasonable accuracy, the results keep on showing that the true states are always inside the estimated bounds and the interval estimation converge to a box whose width depends itself on those measurements error bounds.

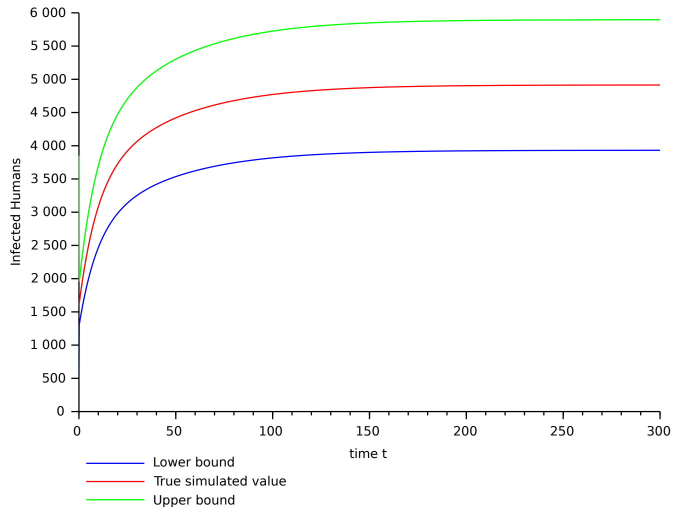

Figure 17: Measures of infected humans population size $X_2(t)$ (green curve) and its lower (red) and upper (blue) bounds estimated with interval observer (16) in presence of uncertainty on the measurements with $b = \pm 20\%\, y_m(\infty)$.

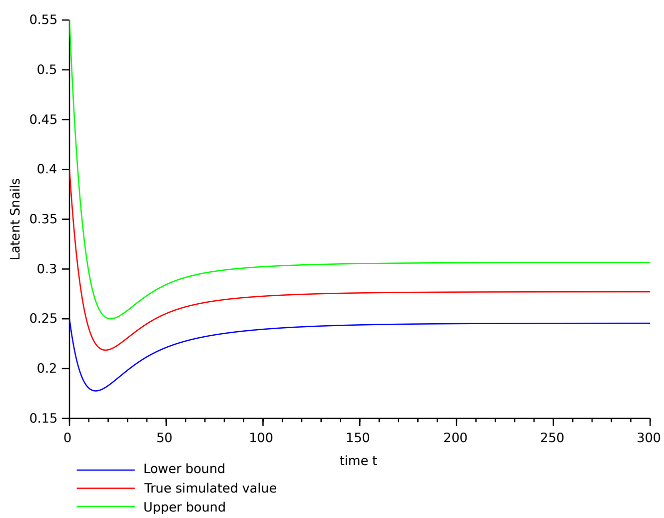

Figure 18: Measures of latent snails population size $x_4(t)$ (green curve) and its lower (red) and upper (blue) bounds estimated with interval observer (16) in presence of uncertainty on the measurements with $b = \pm 20\%\, y_m(\infty)$.

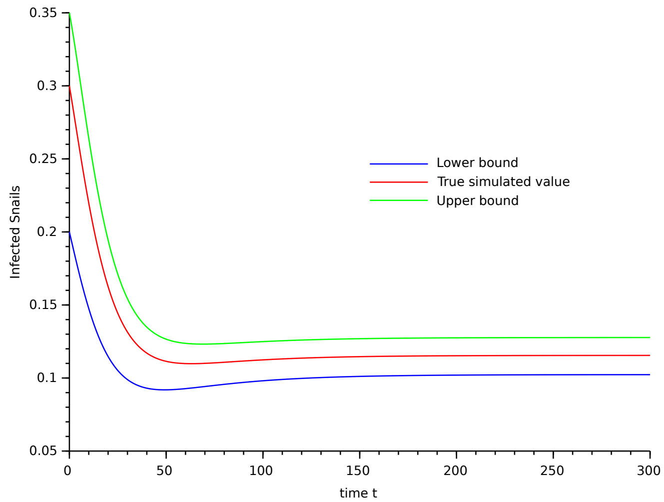

Figure 19: Measures of infected snails population size $x_5(t)$ (green curve) and its lower (red) and upper (blue) bounds estimated with interval observer (16) in presence of uncertainty on the measurements with $b = \pm 20\%\, y_m(\infty)$.

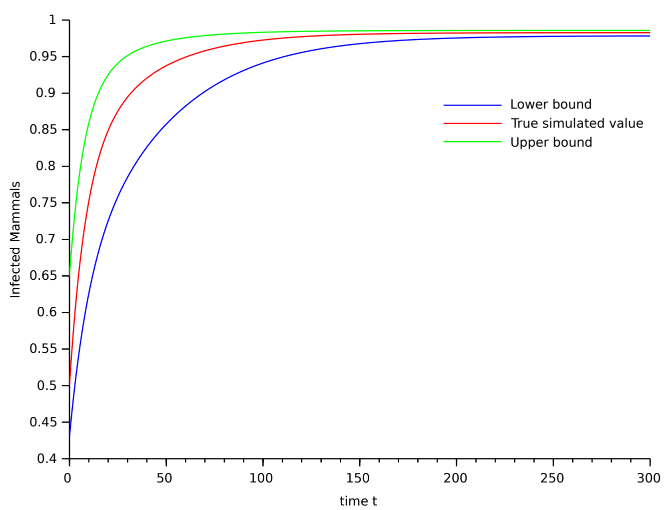

Figure 20: Measures of infected mammals population size $x_7(t)$ (green curve) and its lower (red) and upper (blue) bounds estimated with interval observer (16) in presence of uncertainty on the measurements with $b = \pm 20\%\, y_m(\infty)$.

7. Conclusion

In this paper some nonlinear observers are designed for a mathematical model describing the propagation of schistosomiasis infection among humains, snails and mammalian populations. These observers allow to dynamically estimate the total population size of snails and mammalians (susceptibles as well as infected) using the size of infected humains which is the only available information. More precisely a high-gain observer, a simple nonlinear observer and an interval observer are developed. To test the effectiveness of these observers was tested by performing some numerical simulations that show that the estimates $\hat{x}(t)$ delivered by the various observers converge quite fast to the true states $x(t)$. The convergence of the high-gain observer is the fastest one when the measurements are not corrupted by noise. On the other hand, in the case of noisy measurements the estimates delivered by the high-gain observer are not accurate anymore, while the simple observer gives good estimates with a good convergence rate. Indeed, the high-gain observer is very sensitive to data noise while our simple observer is more robust.

Both the high-gain and the simple observers assume that all the model parameters are precisely known. For the case where the parameters are only known to belong to some bounded intervals, we designed an interval observer to cope with these uncertainties in the model. This interval observer has very good convergence properties and correctly predicts the dynamic bounds for the unmeasured variables even when measurements are corrupted by noise.

Appendix A Proof of Proposition 4.1: the positivity of l_i, $i = 1, \ldots, 3$

Here we shall prove that the derivative of $V(e)$ is negative definite. Thanks to Gauss-Lagrange algorithm, $\dot{V}(e)$ can be written as follows:

$$\dot{V}(e) = -\Big((1+a_2)\ (e_2 + F_2(e_4,e_5))^2 + l_1\ (e_4 + F_4(e_5,e_7))^2 + l_2\ (e_5 + F_5(e_7))^2 + l_3\, e_7^2\Big).$$

where:

$$l_1 \;=\; 1 + a_4 - \frac{{b_{24}}^2}{4(1+\ a_2)},\quad l_2 \;=\; 1 - \frac{{b_{25}}^2}{4(1+\ a_2)} - \frac{\left(\dfrac{b_{24}\, b_{25}}{4(1+\ a_2)} - \dfrac{b_{45}}{2}\right)^2}{1 + a_4 - \dfrac{b_{24}^2}{4(1+\ a_2)}} \text{ and}$$

$$l_3 = 1 + a_7 - \frac{b_{47}^2}{4\left(1 +\ a_4 - \dfrac{{b_{24}}^2}{4(1+\ a_2)}\right)} - \frac{\left(\dfrac{\left(\dfrac{b_{24}b_{25}}{4+4\,a_2} - \dfrac{b_{45}}{2}\right) b_{47}}{2\left(1 + a_4 - \dfrac{{b_{24}}^2}{4(1+a_2)}\right)} + \dfrac{b_{57}}{2}\right)^2}{1 - \dfrac{{b_{25}}^2}{4+4\,a_2} - \dfrac{\left(\dfrac{b_{24}b_{25}}{4+4\,a_2} - \dfrac{b_{45}}{2}\right)^2}{\left(1 + a_4 - \dfrac{{b_{24}}^2}{4(1+\ a_2)}\right)}}.$$

And:

$$F_2(e_4,e_5) = \frac{b_{24}}{2\,(a_2+1)} e_4 - \frac{b_{25}}{2\,(a_2+1)}\, e_5,$$

$$\begin{aligned}
F_4(e_5,e_7) \;&=\; \left(\frac{b_{24}\, b_{25}}{4(a_2+1)\left(-\dfrac{b_{24}^2}{4(a_2+1)} + a_4 + 1\right)} - \frac{b_{45}}{2\left(-\dfrac{b_{24}^2}{4(a_2+1)} + a_4 + 1\right)}\right) e_5 \\
&+\; \frac{b_{47}}{2\left(-\dfrac{b_{24}^2}{4(a_2+1)} + a_4 + 1\right)}\, e_7 \\
&=\; \left(\frac{b_{24}\, b_{25}}{4\,(a_2+1)\, l_1} - \frac{b_{45}}{2\, l_1}\right) e_5 + \frac{b_{47}}{2\, l_1}\, e_7,
\end{aligned}$$

$$F_5(e_7) = \frac{-\dfrac{b_{47}\left(\dfrac{b_{24}\,b_{25}}{4\,(a_2+1)} - \dfrac{b_{45}}{2}\right)}{2\left(-\dfrac{b_{24}^2}{4(a_2+1)} + a_4 + 1\right)} - \dfrac{b_{57}}{2}}{-\dfrac{\left(\dfrac{b_{24}b_{25}}{4(a_2+1)} - \dfrac{b_{45}}{2}\right)^2}{-\dfrac{b_{24}^2}{4(a_2+1)} + a_4 + 1} - \dfrac{b_{25}^2}{4(a_2+1)} + 1}\, e_7$$

$$= \frac{1}{l_2}\left(-\frac{b_{47}\left(\dfrac{b_{24}\,b_{25}}{4\,(a_2+1)} - \dfrac{b_{45}}{2}\right)}{2\,l_1} - \frac{b_{57}}{2}\right) e_7.$$

Let us prove that l_1, l_2 and l_3 are positives with assumption 4.1:

We recall the following quantities:

$$a_2 = \frac{x_5 N_S t_{15}}{L_1 + r_{12}};\;\; a_4 = \frac{X_2 t_{32} + x_7 N_M t_{37}}{b_3 + r_{54}};\;\;\; a_7 = \frac{x_5\, N_S t_{65}}{b_6};$$

$$b_{24} = \frac{(-1+\hat{x}_4+\hat{x}_5)\,t_{32}}{b_3+r_{54}};\;\; b_{45} = \frac{r_{54}}{b_3} - \frac{X_2 t_{32} + x_7 N_M t_{37}}{b_3 + r_{54}};$$

$$b_{47} = \frac{(-1+\hat{x}_4+\hat{x}_5) N_M\, t_{37}}{b_3+r_{54}};\;\; b_{57} = \frac{(1-\hat{x}_7)\, N_S t_{65}}{b_7};\;\; b_{25} = \frac{\left(-\hat{X}_2 + N_H\right) N_S t_{15}}{L_1 + r_{12}}.$$

We have

$$a_7 \leq \frac{N_S t_{65}}{b_7};\;\; -\frac{N_M t_{37}}{b_3 + r_{54}} \leq b_{47} \leq \frac{N_M t_{37}}{b_3 + r_{54}};\;\; -\frac{N_S t_{65}}{b_7} \leq b_{57} \leq \frac{N_S t_{65}}{b_7};$$

$$\frac{r_{54}}{2\,b_3} - \frac{1}{2}\frac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}} \leq \frac{1}{2}\, b_{45} \leq \frac{1}{2}\,\frac{r_{54}}{b_3};\;\; 0 \leq \frac{{b_{25}}^2}{4(1+a_2)} \leq \frac{1}{4}\left(\frac{N_H\, N_S\, t_{15}}{L_1 + r_{12}}\right)^2$$

$$4 \leq 4(1+a_2) \leq 4(1+\frac{N_S t_{15}}{L_1 + r_{12}}) \;\Rightarrow \frac{1}{4}\left(1 + \frac{N_S t_{15}}{L_1 + r_{12}}\right)^{-1} \leq 4\,(1+\,a_2)^{-1} \leq \frac{1}{4};$$

$$-\frac{1}{2}\,\frac{r_{54}}{b_3} \leq -\frac{b_{45}}{2} \leq -\frac{1}{2}\;\frac{r_{54}}{b_3} + \frac{1}{2}\;\frac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}}$$

$$1 \leq 1 + a_4 \leq 1 + \frac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}}$$

$$0 \leq \frac{{b_{24}}^2}{4(1+a_2)} \leq \frac{1}{4} \Rightarrow -\frac{1}{4} \leq -\frac{{b_{24}}^2}{4 + 4\,a_2} \leq 0$$

$$\frac{3}{4} \leq 1 + a_4 - \frac{{b_{24}}^2}{4(1+a_2)} \leq 1 + \frac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}}$$

$$-\frac{1}{4}\;\frac{N_H N_S t_{15} t_{32}}{(L_1 + r_{12})\,(b_3 + r_{54})} \leq \frac{b_{24}\,b_{25}}{4(1+a_2)} \leq \frac{1}{4}\;\frac{N_H N_S t_{15} t_{32}}{(L_1 + r_{12})\,(b_3 + r_{54})}$$

$$\frac{{b_{47}}^2}{4\left(1 + a_4 - \dfrac{{b_{24}}^2}{4(1+a_2)}\right)} \leq \frac{1}{3}\left(\frac{N_M\, t_{37}}{b_3 + r_{54}}\right)^2$$

$$-\frac{1}{2}\,\frac{r_{54}}{b_3} \;\leq\; \frac{b_{24} b_{25}}{4(1+a_2)} - \frac{1}{2} b_{45} \;\leq\; \frac{1}{4}\,\frac{N_H N_S t_{15} t_{32}}{L_1\,(b_3 + r_{54}) + r_{12}\,(b_3 + r_{54})} - \frac{1}{2}\,\frac{r_{54}}{b_3} + \frac{1}{2}\,\frac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}}$$

$$-\frac{\dfrac{1}{2}\,\dfrac{r_{54}}{b_3}}{\left(1 + \dfrac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}}\right)} \leq \frac{\dfrac{b_{24} b_{25}}{4(1+a_2)} - \dfrac{b_{45}}{2}}{\left(1 + a_4 - \dfrac{{b_{24}}^2}{4(1+a_2)}\right)} \leq \frac{4}{3}\,\frac{N_H N_S t_{15} t_{32}}{4\,(b_3 + r_{54})\,(L_1 + r_{12})} - \frac{r_{54}}{2\,b_3} + \frac{N_H t_{32} + N_M t_{37}}{2(b_3 + r_{54})}$$

$$\frac{\left(\dfrac{b_{24} b_{25}}{4(1+a_2)} - \dfrac{b_{45}}{2}\right)^2}{\left(1 + a_4 - \dfrac{{b_{24}}^2}{4(1+a_2)}\right)} \leq \frac{1}{3}\left(\frac{1}{2}\,\frac{N_H N_S t_{15}}{L_1 + r_{12}} + \frac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}} - \frac{r_{54}}{b_3}\right)^2$$

Since $\dfrac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}} < 1$, we have $\dfrac{t_{32}}{r_{54} + b_3} < 1$, we get

$$l_1 = 1 + a_4 - \frac{{b_{24}}^2}{4(1+a_2)} \geq 1 - \frac{1}{4}\left(\frac{t_{32}}{b_3 + r_{54}}\right)^2 > 0$$

$\star$ first case: Iff $b_{45} \geq 0$

We define
$X := \dfrac{N_H N_S t_{15} t_{32}}{(L_1 + r_{12})\,(b_3 + r_{54})}$ and $Y := \dfrac{r_{54}}{b_3} - \dfrac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}}$

We choose $L_1 \geq 0$ so that $X \leq 1/2$.
we get
$-\left(\dfrac{1}{2}\dfrac{r_{54}}{b_3} + \dfrac{X}{4}\right) \leq V_1 := \dfrac{b_{24}\,b_{25}}{4(1+a_2)} - \dfrac{1}{2}b_{45} \leq 0 \Rightarrow V_1^2 \leq \dfrac{1}{4}\left(\dfrac{r_{54}}{b_3} + \dfrac{X}{2}\right)^2$

$$
\begin{aligned}
l_2 = &\quad 1 - \frac{{b_{25}}^2}{4(1+a_2)} - \frac{\left(\dfrac{b_{24}b_{25}}{4(1+a_2)} - \dfrac{b_{45}}{2}\right)^2}{\left(1 + a_4 - \dfrac{{b_{24}}^2}{4(1+a_2)}\right)} \\
l_2 \geq &\quad 1 - \frac{1}{4}\left(\frac{N_H\,N_S\,t_{15}}{L_1 + r_{12}}\right)^2 - \frac{1}{3}\left(\frac{r_{54}}{b_3} + \frac{X}{2}\right)^2 \\
\geq &\qquad \frac{3}{4} > 0
\end{aligned}
$$

and since $b_{47} \leq 0$

$V_2 := \dfrac{V_1\,b_{47}}{2\left(1 + a_4 - \dfrac{{b_{24}}^2}{4(1+a_2)}\right)} + \dfrac{b_{57}}{2} \leq \dfrac{V_1\,b_{47}}{2\,\dfrac{3}{4}} + \dfrac{b_{57}}{2} \leq \dfrac{2}{3}\left(\dfrac{X}{4} + \dfrac{r_{54}}{2\,b_3}\right)\dfrac{N_M t_{37}}{b_3 + r_{54}} + \dfrac{N_S t_{65}}{2\,b_7}$

$l_3 \geq 1 + a_7 - \dfrac{1}{3}\left(\dfrac{N_M\,t_{37}}{b_3 + r_{54}}\right)^2 - \dfrac{V_2^2}{l_2}.$

Since $\dfrac{N_M\,t_{37}}{b_3 + r_{54}} \leq \dfrac{N_H t_{32} + N_M t_{37}}{b_3 + r_{54}} \leq \dfrac{r_{54}}{b_3}$

we get
$l_3 \geq 1 + a_7 - \dfrac{1}{3}\left(\dfrac{r_{54}}{b_3}\right)^2 - \dfrac{V_2^2}{l_2}$

$l_3 \geq 1 + a_7 - \dfrac{1}{3}\left(\dfrac{r_{54}}{b_3}\right)^2 - 4\,\left(\dfrac{2}{3}\left(\dfrac{X}{4} + \dfrac{r_{54}}{2\,b_3}\right)\dfrac{r_{54}}{b_3} + \dfrac{N_S t_{65}}{2\,b_7}\right)^2$

$l_3 \geq a_7 + \dfrac{11}{12} - \dfrac{25}{48} > 0$

$\star$ Second case: $\quad b_{45} \leq 0 \Rightarrow Y \leq b_{45} \leq 0$

We get

$$-\frac{1}{2}\frac{r_{54}}{b_3} \leq V_1 := \frac{b_{24}\, b_{25}}{4(1+a_2)} - \frac{1}{2} b_{45} \leq \frac{X}{4} - \frac{Y}{2}$$

If $|Y| \leq \dfrac{r_{54}}{b_3}$ then

$$|V_1| \leq |\frac{b_{24}\, b_{25}}{4(1+a_2)}| + |-\frac{1}{2} b_{45}| \leq \frac{X}{4} + \frac{r_{54}}{2\, b_3}$$

It follows that $0 \leq V_1^2 \leq \left(\dfrac{X}{4} + \dfrac{r_{54}}{2\, b_3}\right)^2$

So

$$l_2 \geq 1 - \frac{1}{4}\left(\frac{N_H\, N_S\, t_{15}}{L_1 + r_{12}}\right)^2 - \frac{1}{3}\left(\frac{X}{4} + \frac{r_{54}}{2\, b_3}\right)^2$$

$$l_2 \geq \frac{57}{64} > 0$$

$$|V_2| \leq |\frac{V_1\, b_{47}}{2\,\frac{3}{4}}| + |\frac{b_{57}}{2}| \leq \frac{2}{3}\left(\frac{N_M\, t_{37}}{b_3 + r_{54}}\right)\left(\frac{X}{4} + \frac{r_{54}}{2\, b_3}\right) + \frac{N_S t_{65}}{2\, b_7}$$

$$l_3 \geq 1 + a_7 - \frac{1}{3}\left(\frac{N_M t_{37}}{b_3 + r_{54}}\right)^2 - \frac{V_2^2}{l_2}.$$

Since $-\dfrac{r_{54}}{b_3} \leq Y \Rightarrow \dfrac{N_M t_{37}}{b_3 + r_{54}} \leq 2\dfrac{r_{54}}{b_3}$, we obtain $l_3 \geq 1 + a_7 - \dfrac{1}{3} - \dfrac{36}{57} > 0.$

If $|Y| \geq \dfrac{r_{54}}{b_3}$ then

$$|V_1| \leq |\frac{b_{24}\, b_{25}}{4(1+a_2)}| + |-\frac{1}{2} b_{45}| \leq \frac{X}{4} + \frac{Y}{2}$$

It follows that $0 \leq V_1^2 \leq \left(\dfrac{X}{4} + \dfrac{Y}{2}\right)^2$

So

$$l_2 \geq 1 - \frac{1}{4}\left(\frac{N_H\, N_S\, t_{15}}{L_1 + r_{12}}\right)^2 - \frac{1}{3}\left(\frac{r_{54}}{b_3}\right)^2$$

$$l_2 \geq \frac{41}{48} > 0$$

$$|V_2| \leq |\frac{V_1\, b_{47}}{2\,\frac{3}{4}}| + |\frac{b_{57}}{2}| \leq \frac{2}{3}\left(\frac{N_M\, t_{37}}{b_3 + r_{54}}\right)\left(\frac{X}{4} + \frac{Y}{2}\right) + \frac{N_S t_{65}}{2\, b_7}$$

$$V_2^2 \leq \left(\frac{V_1\, b_{47}}{2\,\dfrac{3}{4}} + \frac{b_{57}}{2} \right)^2 \leq \left(\frac{2}{3} \left(\frac{N_M\, t_{37}}{b_3 + r_{54}} \right) \left(\frac{X}{4} + \frac{Y}{2} \right) + \frac{N_S t_{65}}{2\, b_7} \right)^2$$

So

$$l_3 \geq 1 + a_7 - \frac{1}{3} \left(\frac{N_M t_{37}}{b_3 + r_{54}} \right)^2 - \frac{V_2^2}{l_2}\ l_3 \geq 1 + a_7 - \frac{1}{12} - \frac{400}{1107} > 0.$$

And finally we give here the full expression of $R(e, \hat{x}) = \begin{pmatrix} R_{1,1} & R_{1,2} \\ R_{2,1} & R_{2,2} \end{pmatrix}$ where:

$$R_{1,1} = \begin{pmatrix} -\frac{1}{2}(e_5 + 2\hat{x}_5) N_S t_{15} & 0 \\ -\frac{1}{2}(e_4 + e_5 + 2(\hat{x}_4 + \hat{x}_5 - 1)) t_{32} & \frac{1}{2}\left((-e_2 - 2\hat{X}_2) t_{32} - (e_7 + 2\hat{x}_7) N_M t_{37} \right) \end{pmatrix}$$

$$R_{1,2} = \begin{pmatrix} -\frac{1}{2}\left(e_2 + 2\hat{X}_2 - 2N_H \right) N_S t_{15} & 0 \\ \frac{1}{2}\left((-e_2 - 2\hat{X}_2) t_{32} - (e_7 + 2\hat{x}_7) N_M t_{37} \right) & -\frac{1}{2}(e_4 + e_5 + 2(\hat{x}_4 + \hat{x}_5 - 1)) N_M t_{37} \end{pmatrix}$$

$$R_{2,1} = \begin{pmatrix} 0 & r_{54} \\ 0 & 0 \end{pmatrix}; \quad R_{2,2} = \begin{pmatrix} 0 & 0 \\ -\frac{1}{2}(e_7 + 2\hat{x}_7 - 2) N_S t_{65} & -\frac{1}{2}(e_5 + 2\hat{x}_5) N_S t_{65} \end{pmatrix}.$$

Appendix B Existence of a finite positive real number M satisfying condition (13)

We have $p = (t_{15}, t_{32}, t_{37}, t_{65})$ where $t_{15} \in [t_{15}^-, t_{15}^+]$, $t_{32} \in [t_{32}^-, t_{32}^+]$, $t_{37} \in [t_{37}^-, t_{37}^+]$, $t_{65} \in [t_{65}^-, t_{65}^+]$, and $x = (X_2, x_4, x_5, x_7) \in \mathcal{D} = [0, N_H] \times [0,1] \times [0,1] \times [0,1]$.

We recall the following vectors :

$$\psi = \begin{pmatrix} t_{15}\,(N_H - X_2)\, N_S\, x_5 \\ (t_{32} X_2 + t_{37} N_M x_7)\,(1 - x_4 - x_5) \\ 0 \\ t_{65} N_{\mathrm{S}} x_5\,(1 - x_7) \end{pmatrix},$$

$$\psi^- = \begin{pmatrix} t_{15}^-\,(N_H - X_2)\, N_S\, x_5 \\ \left(t_{32}^- X_2 + t_{37}^- N_M x_7\right)(1 - x_4 - x_5) \\ 0 \\ t_{65}^- N_{\mathrm{S}} x_5\,(1 - x_7) \end{pmatrix}$$

and

$$\psi^+ = \begin{pmatrix} t_{15}^+\,(N_H - X_2)\, N_S\, x_5 \\ \left(t_{32}^+ X_2 + t_{37}^+ N_M x_7\right)(1 - x_4 - x_5) \\ 0 \\ t_{65}^+ N_{\mathrm{S}} x_5\,(1 - x_7) \end{pmatrix}.$$

We have the following inequalities :

$$t_{15}^{-}\left(N_H - X_2\right) N_S\, x_5 \leq t_{15}\left(N_H - X_2\right) N_S\, x_5 \leq t_{15}^{+}\left(N_H - X_2\right) N_S\, x_5,$$

as we have $(N_H - X_2) \geq 0$.

$$\begin{aligned}\left(t_{32}^{-}X_2 + t_{37}^{-}N_M x_7\right)(1 - x_4 - x_5) &\leq \left(t_{32}X_2 + t_{37}N_M x_7\right)(1 - x_4 - x_5) \\ &\leq \left(t_{32}^{+}X_2 + t_{37}^{+}N_M x_7\right)(1 - x_4 - x_5),\end{aligned}$$

since $(1 - x_4 - x_5) \geq 0$.

$$t_{65}^{-}N_{\mathrm{S}}x_5\,(1 - x_7) \leq t_{65}N_{\mathrm{S}}x_5\,(1 - x_7) \leq t_{65}^{+}N_{\mathrm{S}}x_5\,(1 - x_7)\,,\ \text{due to } (1 - x_7) \geq 0.$$

In this manner $\psi^{-} \leq \psi \leq \psi^{+}$ for all $x \in \mathcal{D}$.

Let us prove now the last item of the condition (13). We have

$$\begin{aligned}\mid t_{15}^{-}\left(N_H - X_2\right) N_S\, x_5 - \left(t_{15}^{+}\left(N_H - X_2\right) N_S\, x_5\right) \mid &\leq \left(N_H - X_2\right)\, N_S\, x_5\, \left(t_{15}^{+} - t_{15}^{-}\right) \\ &\leq N_H\, N_S\, \left(t_{15}^{+} - t_{15}^{-}\right).\end{aligned}$$

$$\begin{aligned}&\mid \left(t_{32}^{-}X_2 + t_{37}^{-}N_M x_7\right)(1 - x_4 - x_5) - \left(\left(t_{32}^{+}X_2 + t_{37}^{+}N_M x_7\right)(1 - x_4 - x_5)\right) \mid \\ &\qquad \leq (1 - x_4 - x_5) \mid x_2\left(t_{32}^{-} - t_{32}^{+}\right) + N_M x_7\left(t_{37}^{-} - t_{37}^{+}\right) \mid \\ &\qquad \leq x_2\ \mid t_{32}^{-} - t_{32}^{+} \mid + N_M\, x_7 \mid t_{37}^{-} - t_{37}^{+} \mid \\ &\qquad \leq \left(t_{32}^{+} - t_{32}^{-}\right) + N_M\, \left(t_{37}^{+} - t_{37}^{-}\right).\end{aligned}$$

$$\begin{aligned}\mid t_{65}^{-}N_{\mathrm{S}}x_5\,(1 - x_7) - \left(t_{65}^{+}N_{\mathrm{S}}x_5\,(1 - x_7)\right) \mid &\leq N_S\, x_5(1 - x_7) \mid t_{65}^{-} - t_{65}^{+} \mid \\ &\leq N_S\, \left(t_{65}^{+} - t_{65}^{-}\right).\end{aligned}$$

Consequently we set

$$M := \max\{N_H\, N_S\, \left(t_{15}^{+} - t_{15}^{-}\right), \left(t_{32}^{+} - t_{32}^{-}\right) + N_M\, \left(t_{37}^{+} - t_{37}^{-}\right), N_S\, \left(t_{65}^{+} - t_{65}^{-}\right)\}.$$

References

[1] W. H. Organization, UNAIDS., Prevention and Control of Schistosomiasis and Soil-Transmitted Helminthiasis: Report of a WHO Expert Committee, World Health Organization, 2002.

[2] C.-T. Tang, M.-K. Lu, Y. Guo, Y.-N. Wang, J.-Y. Peng, W.-B. Wu, W.-H. Li, B. C. Weimer, D. Chen, Development of larval Schistosoma japonicum blocked in Oncomelania hupensis by pre-infection with larval Exorchis sp., The Journal of parasitology 95 (6) (2009) 1321–5. doi:10.1645/GE-2055.1.

[3] D. Luenberger, An introduction to observers, IEEE Transactions on Automatic Control 16 (6) (1971) 596–602. doi:10.1109/TAC.1971.1099826.

[4] A. J. Krener, A. Isidori, Linearization by output injection and nonlinear observers, Systems & Control Letters 3 (1) (1983) 47–52. doi:10.1016/0167-6911(83)90037-3.

[5] A. J. Krener, W. Respondek, Nonlinear Observers with Linearizable Error Dynamics, SIAM Journal on Control and Optimization 23 (2) (1985) 197–216. doi:10.1137/0323016.

[6] H. Hammouri, J. P. Gauthier, Bilinearization up to output injection, Systems & Control Letters 11 (2) (1988) 139–149. doi:10.1016/0167-6911(88)90088-6.

[7] H. Hammouri, J. P. Gauthier, Global time-varying linearization up to output injection, SIAM Journal on Control and Optimization 30 (6) (1992) 1295–1310. doi:10.1137/0330068.

[8] J. L. Gouzé, M. Moisan, O. Bernard, A simple improvement of interval asymptotic observers for biotechnological processes, Robust Control Design 5 (2006) 119–124. doi:10.3182/20060705-3-FR-2907.00022.

[9] E. J. Allen, H. D. Victory, Modelling and simulation of a schistosomiasis infection with biological control, Acta Tropica 87 (2) (2003) 251–267. doi:10.1016/S0001-706X(03)00065-2.

[10] J. P. Gauthier, H. Hammouri, S. Othman, A simple observer for nonlinear systems applications to bioreactors, IEEE Transactions on Automatic Control 37 (6) (1992) 875–880. doi:10.1109/9.256352.

[11] J. P. Gauthier, H. Hammouri, I. A. K. Kupka, Observers for nonlinear systems, in: Proceedings of the 30th IEEE Conference on Decision and Control, IEEE, 1991, pp. 1483–1489. doi:10.1109/CDC.1991.261648.

[12] J. P. Gauthier, I. A. K. Kupka, Observability and observers for nonlinear systems, SIAM Journal on Control and Optimization 32 (4) (1994) 975–994. doi:10.1137/S0363012991221791.

[13] A. Rapaport, J. L. Gouzé, Parallelotopic and practical observers for nonlinear uncertain systems, International Journal of Control 76 (3) (2003) 237–251. doi:10.1080/0020717031000067457.

[14] N. Meslem, N. Ramdani, Y. Candau, Interval Observers for Uncertain Nonlinear Systems. Application to Bioreactors, in: World Congress, Vol. 17, 2008, pp. 9667–9672. doi:10.3182/20080706-5-KR-1001.01635.

[15] J. L. Gouzé, A. Rapaport, M. Z. Hadj-Sadok, Interval observers for uncertain biological systems, Ecological Modelling 133 (1-2) (2000) 45–56. doi:10.1016/S0304-3800(00)00279-9.

[16] I. Rocha, A. C. A. Veloso, E. C. Ferreira, Design of estimators for specific growth rate control in a fed-batch E. coli fermentation, in: Proceedings 5th Vienna Symposium on Mathematical Modelling, Vol. 2, ARGESIM, 2006, pp. 1–9.

[17] A. Iggidr, Controllability, observability and stability of mathematical models, Encyclopedia of life support systems (EOLSS). Developed under the auspices of the UNESCO, Eolss Publishers, Oxford, UK. http://www. eolss. net. 31.

[18] E. J. McShane, Extension of range of functions, Bulletin of the American Mathematical Society 40 (12) (1934) 837–843. doi:10.1090/S0002-9904-1934-05978-0.

[19] A. Ball, H. K. Khalil, High-gain observers in the presence of measurement noise: A nonlinear gain approach, in: 47th IEEE Conference on Decision and Control, 2008. CDC 2008., IEEE, 2008, pp. 2288–2293. doi:10.1109/CDC.2008.4739408.